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Experimental Sub-structuring for Linear and Nonlinear Connection Dynamics: A Tutorial

Part 1: Linear Joints – Daniel Rixen, TUM

Part 2: Nonlinear Joints – H. Nevzat Özgüven, METU

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Experimental Sub-structuring for Linear and Nonlinear Connection Dynamics: A Tutorial - Part 2: Nonlinear Joints

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Outline

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- **Introduction**
- Quasi-linear Behaviour of Nonlinear Systems
  - Nonlinearity Matrix Concept
  - RCT – Response Controlled Stepped-Sine Testing (Experimental Nonlinear Modal Analysis)
- Nonlinear Structural Coupling
- Nonlinear Structural Decoupling and Nonlinear Joint Identification
- Applications

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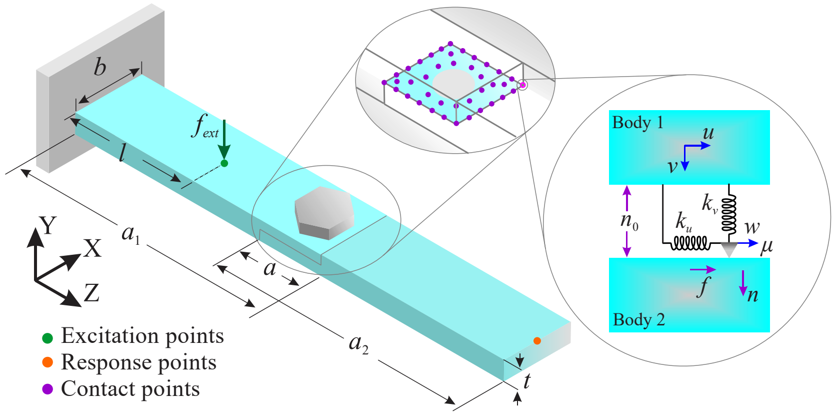
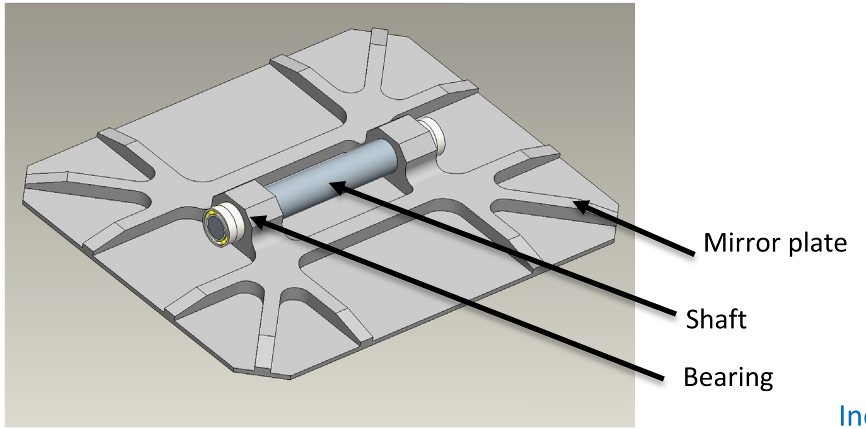
Introduction

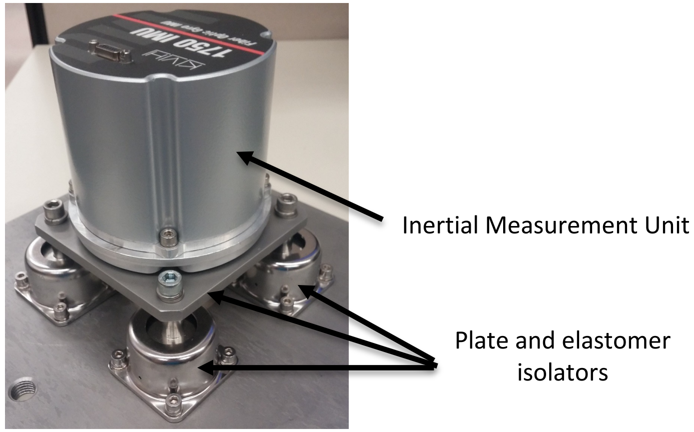
We may have nonlinear connection dynamics in various applications



Bolted connection with nonlinear contact elements



Mirror plate-shaft assembly – Used in land platforms for optical purposes



Inertial Measurement Unit and its mechanical interface plate grounded with elastomer isolators - Used in aerospace platforms

Aircraft-pylon-store assembly

Guided missiles: Damping nonlinearity due to bolted connections

Strong stiffness and damping nonlinearity due to control fin connection

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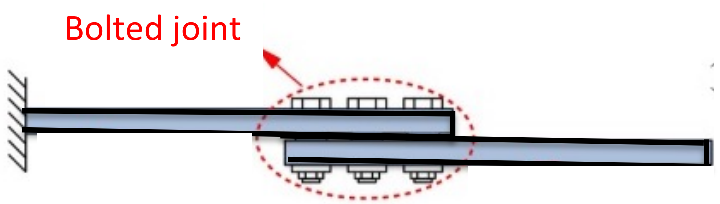
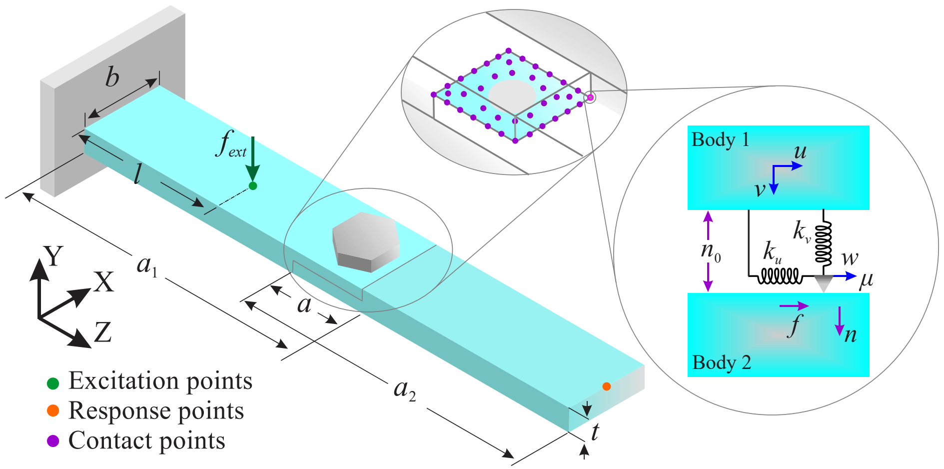
Introduction

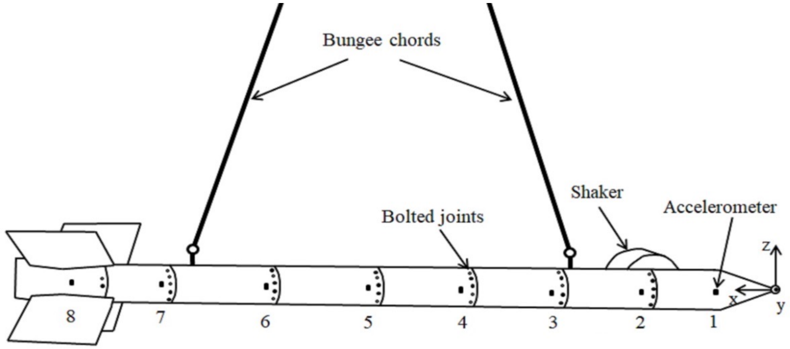


Parametric identification is possible

a) Direct identification

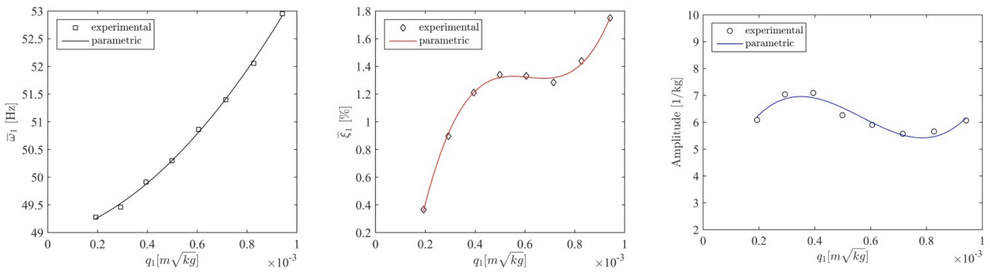
b) Sub-structuring (will be discussed)





Parametric identification of individual nonlinear connections (bolts) is not possible

Obtain a modal model for the whole NL structure (will be discussed)



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Introduction

- Studies on dynamic sub-structuring:
  - for linear systems – Hurty, Craig and Bampton (1960s)
  - for nonlinear systems are more recent (10 - 15 years):
- There are several ongoing studies on experimental dynamic sub-structuring of nonlinear systems.
- Unlike linear sub-structuring methods, available techniques are still not general nor perfect.
- However, some of them have given good results in engineering applications.

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Introduction

This presentation covers frequency-based nonlinear sub-structuring approaches making use of quasi-linear behavior of nonlinear systems

- Quasi-linearization in Analytical Computations
- Quasi-linearization in Experimental Analysis

Assumption: Harmonic excitation yields harmonic response (sub- and super-harmonics are negligible) in the frequency range of interest

Quasi-linearization is not an approximation (unlike linearization)

Based on this concept, it is possible to extend several methods for linear systems to nonlinear cases

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- Introduction
- **Quasi-linear Behaviour of Nonlinear Systems**
  - **Nonlinearity Matrix Concept**
  - **RCT – Response Controlled Stepped-Sine Testing**
- Nonlinear Structural Coupling
- Nonlinear Structural Decoupling and Nonlinear Joint Identification
- Applications

Nonlinearity Matrix Concept

Quasi-linearization in Analytical Computations

Consider a nonlinear MDOF system subjected to a harmonic excitation:

$$\underbrace{\boldsymbol{M\ddot{u}} + \boldsymbol{C\dot{u}} + i\boldsymbol{Hu} + \boldsymbol{Ku}}_{\text{Underlying linear system}} + \underset{\substack{\downarrow \\ \text{Nonlinear internal forces}}}{\boldsymbol{f}_N} = \boldsymbol{f} = \boldsymbol{F}e^{i\omega t}$$

For a harmonic excitation $\boldsymbol{f} = \boldsymbol{F}e^{i\omega t}$ ⇨ response can be assumed to be harmonic: $\boldsymbol{u} = \boldsymbol{U}e^{i\omega t}$

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Nonlinearity Matrix Concept

For harmonic response $\boldsymbol{u} = \boldsymbol{U}e^{i\omega t}$ ⇨ Nonlinear internal forces: $f_N = \boxed{\boldsymbol{F}_N} e^{i\omega t}$

$$\boxed{\boldsymbol{F}_N = \boldsymbol{\Delta}(\boldsymbol{U})\,\boldsymbol{U}}$$

⬅ **Key equation**

$\boldsymbol{\Delta}(\boldsymbol{U})$: **Nonlinearity matrix*** (response level dependent)

Elements of nonlinearity matrix are calculated using Describing Functions

$$\Delta_{kk} = \upsilon_{kk} + \sum_{\substack{j=1 \\ j\neq k}}^{n} \upsilon_{kj}$$

$$\Delta_{kj} = -\upsilon_{kj} \qquad for\ j \neq k$$

$$\upsilon_{rj}(\bar{U}_{rj}) = \frac{i}{\pi \bar{U}_{rj}} \int_0^{2\pi} n_{rj}(\bar{u}_{rj}) e^{-i\tau} d\tau$$

$$\bar{U}_{rj} = U_r - U_j \quad for\ r \neq j$$
$$\bar{U}_{rj} = U_r \quad for\ r = j$$

*Tanrıkulu, Ö., Kuran, B., Özgüven, H. N. and Imregün, M., “Forced Harmonic Response Analysis of Nonlinear Structures Using Describing Functions”, ***AIAA Journal***, *1993.*

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Nonlinearity Matrix Concept

$$\boldsymbol{M\ddot{u}} + \boldsymbol{C\dot{u}} + i\boldsymbol{Hu} + \boldsymbol{Ku} + \boldsymbol{f}_N = \boldsymbol{f} = \boldsymbol{F}e^{i\omega t}$$

Equation of motion in time domain

$$\boxed{-\omega^2\boldsymbol{MU} + i\omega\boldsymbol{CU} + i\boldsymbol{HU} + \boldsymbol{KU}} + \boxed{\boldsymbol{F}_N} = \boldsymbol{F}$$

Equation of motion in frequency domain

Underlying linear system

Amplitude of nonlinear internal forces

$$\boldsymbol{F}_N = \boldsymbol{\Delta}(\boldsymbol{U})\,\boldsymbol{U}$$

$$-\omega^2\boldsymbol{MU} + i\omega\boldsymbol{CU} + i\boldsymbol{HU} + \boldsymbol{KU} + \boldsymbol{\Delta}(\boldsymbol{U})\,\boldsymbol{U} = \boldsymbol{F}$$

$\boldsymbol{\Delta}(\boldsymbol{U})$ acts like an equivalent stiffness (and damping) matrix

$$\boxed{\boldsymbol{Y}_{NL} = \left(-\omega^2\boldsymbol{M} + i\omega\boldsymbol{C} + i\boldsymbol{H} + [\boldsymbol{K} + \boldsymbol{\Delta}(\boldsymbol{U})]\right)^{-1}}$$

- It is response level dependent
- Therefore computations require iteration

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Response Controlled Stepped-Sine Testing*

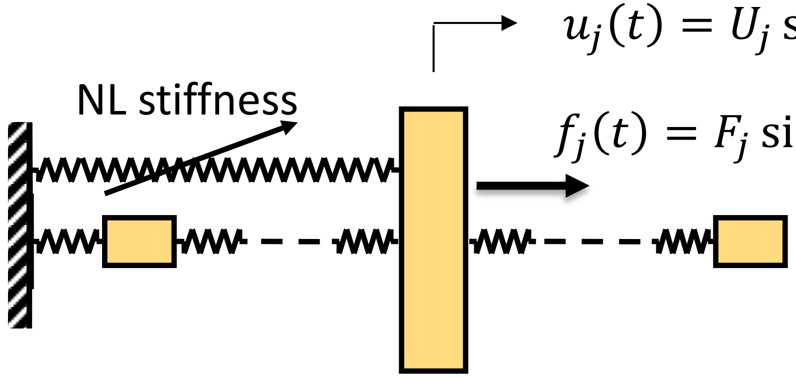
Quasi-linearization in Experimental Analysis

Consider a system with a single NL element

$$u_j(t) = U_j \sin(\omega t - \emptyset)$$

NL stiffness

$$f_j(t) = F_j \sin(\omega t)$$

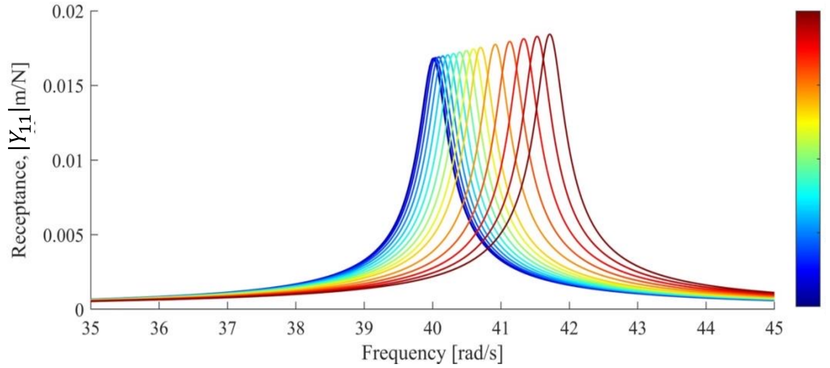


Keep U_j constant during stepped-sine test (frequency sweep around resonance)

NL internal force will be constant during frequency sweep

System will behave quasi-linear

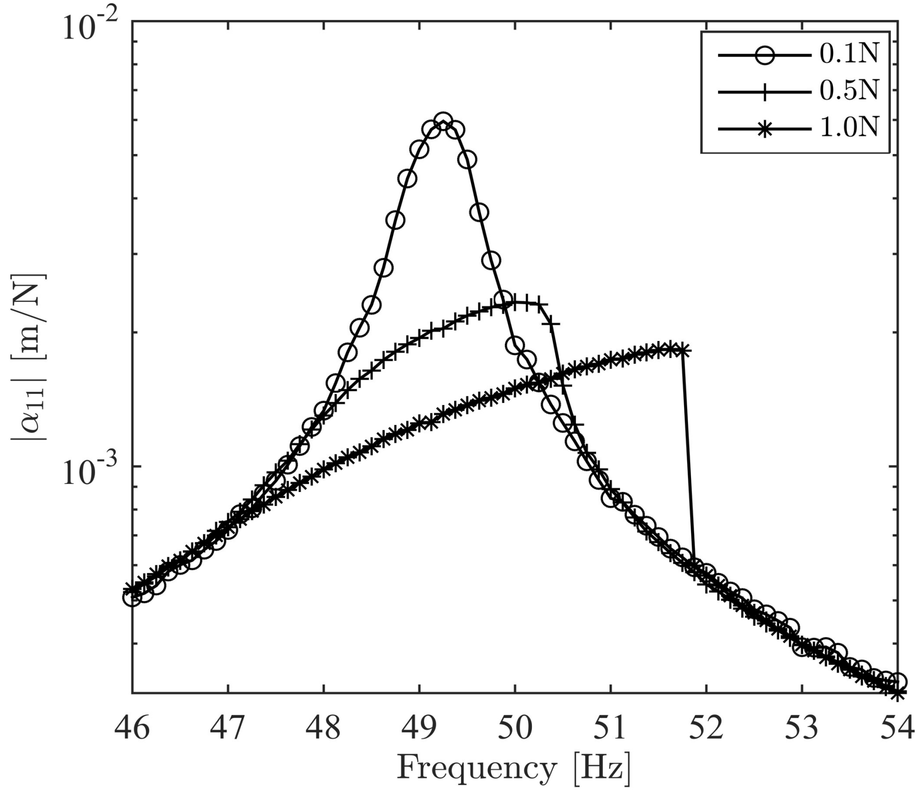
Each FRF will correspond to a different response level U_j



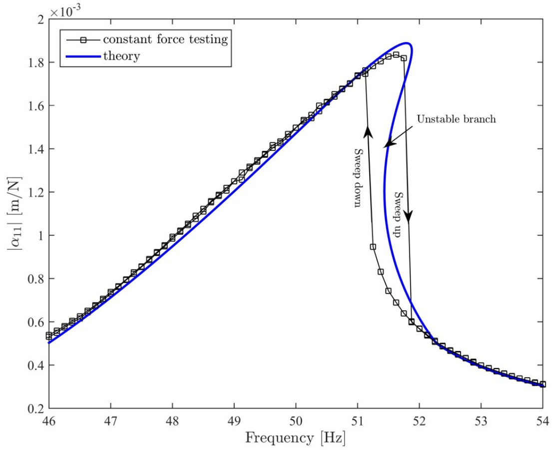
*Arslan, Ö., Aykan, M. and Özgüven, H. N., “Parametric Identification of Structural Nonlinearities from Measured Frequency Response Data”, ***Mechanical Systems and Signal Processing***, 2011.*

Response Controlled Stepped-Sine Testing

If a classical constant-force amplitude stepped-sine test were made:

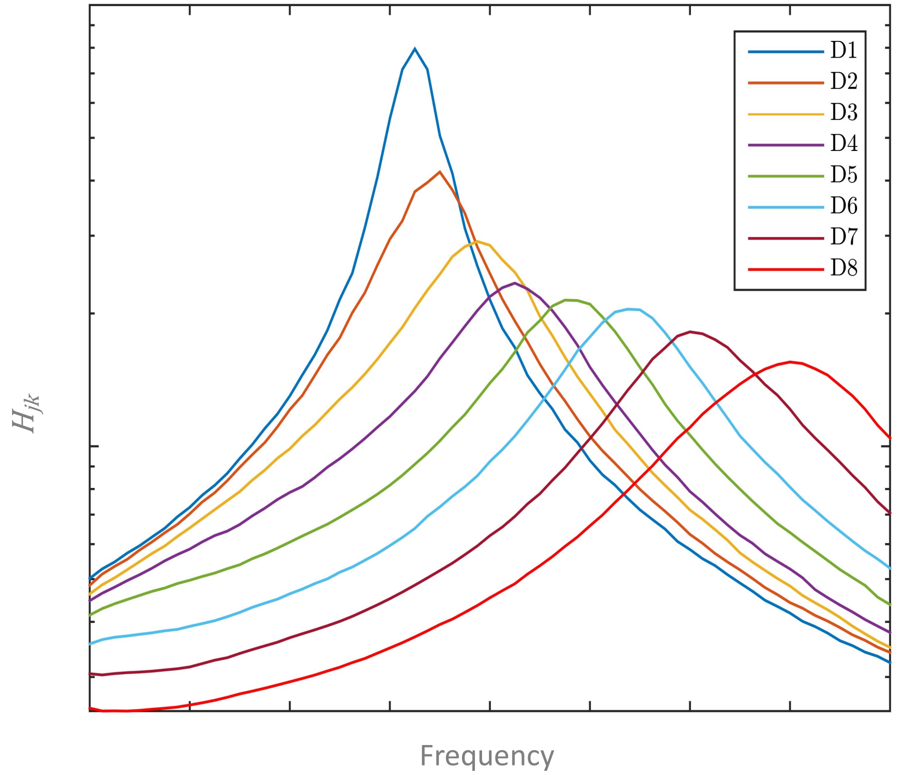


- NL FRFs would be obtained
- Different FRF for each force level
- Identification of modal parameters is not possible
- When jump occurs, it is not possible to capture unstable orbits

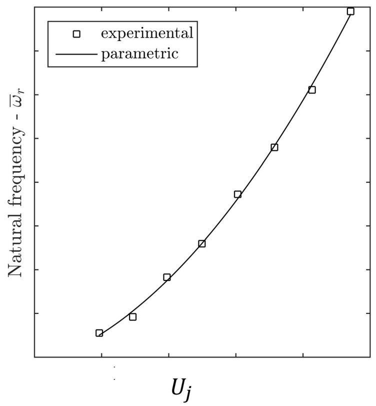


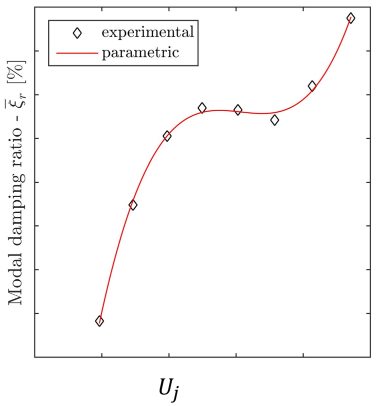
Experimental Nonlinear Modal Analysis

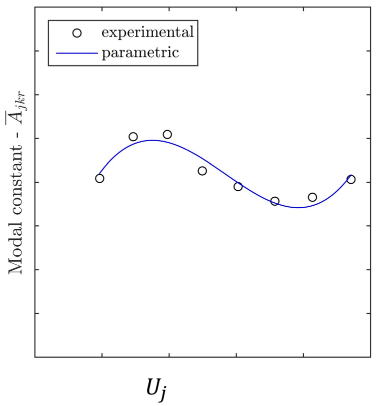
Quasi-linear FRFs of a nonlinear system



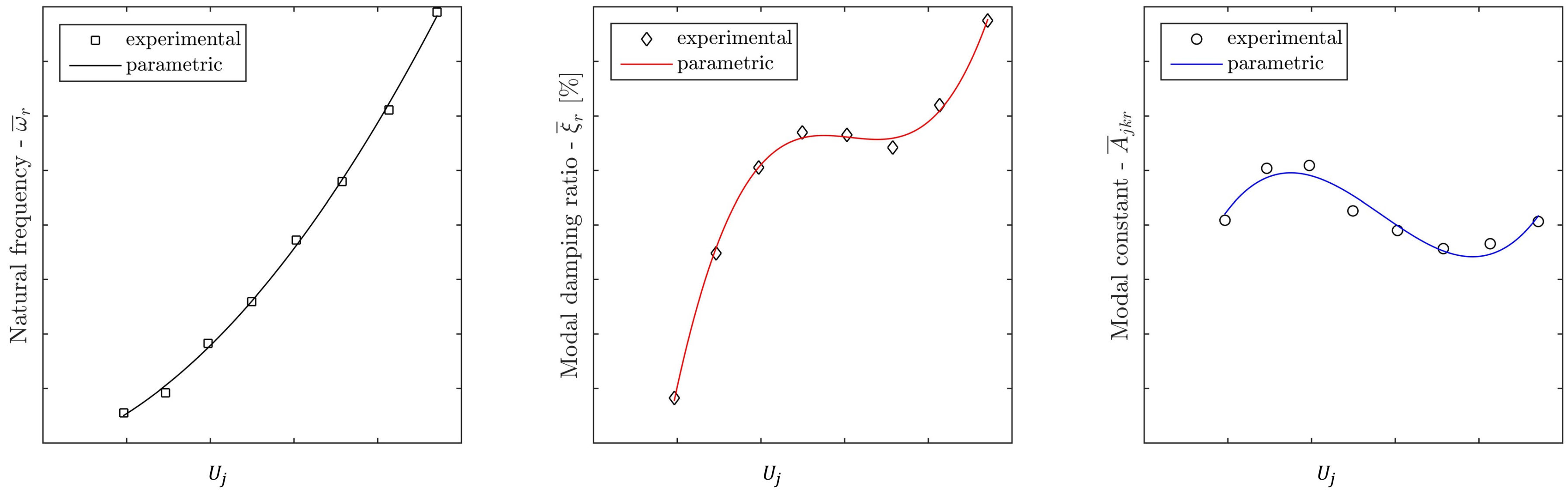
- A set of modal properties can be identified from each FRF
- Each FRF will correspond to a different response level U_j
- Each set will be valid only for the response level at which the test is made
- Then we can plot the variation of each modal property with the response level







Experimental Nonlinear Modal Analysis



$$Y_{jk}(\omega, U_j) = \frac{\bar{\phi}_{jr}(U_j)\bar{\phi}_{kr}(U_j)}{\overline{\omega}_r^2(U_j) - \omega^2 + i2\bar{\xi}_r(U_j)\omega\overline{\omega}_r(U_j)}$$

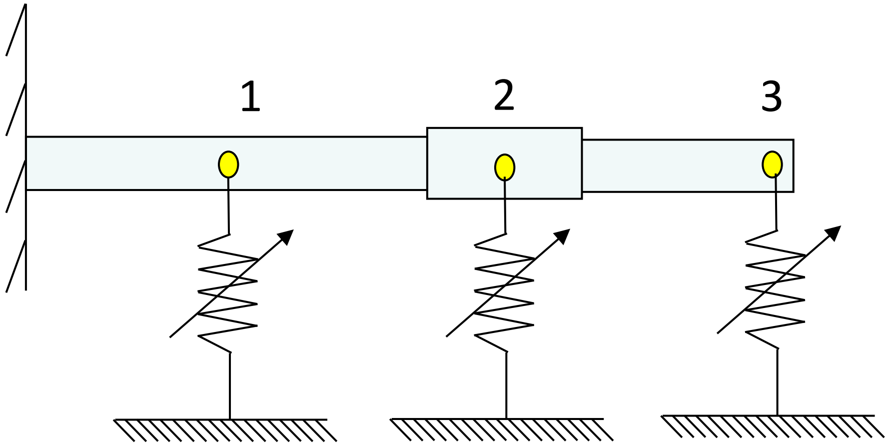
$$U_j = Y_{jk}F_k$$

(Iteration is required)

Response Controlled Stepped-Sine Testing

What if we have more than one nonlinear element?

What if we have distributed nonlinearity?



- If we keep U_1 constant during stepped-sine test, can we guarantee that U_2 and U_3 will remain constant as well?
- At first glance it does not seem to be so!

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Single Nonlinear Mode Theory

- According to the **Single Nonlinear Mode Theory***, if we have well separated modes and no internal resonance, near-resonant vibration response can be represented as follows:

$$\boldsymbol{U} = q_r \boxed{\boldsymbol{\Phi}_r(q_r)}$$

NNM is a **function** of response level

Modal Amplitude

Nonlinear Normal Mode (NNM)

**Szemplinska-Stupnicka, W., "The Modified Single Mode Method in the Investigation of the Resonant Vibration of Nonlinear Systems", Journal of Sound and Vibration, v. 63(4), pp. 475-489, 1979.*

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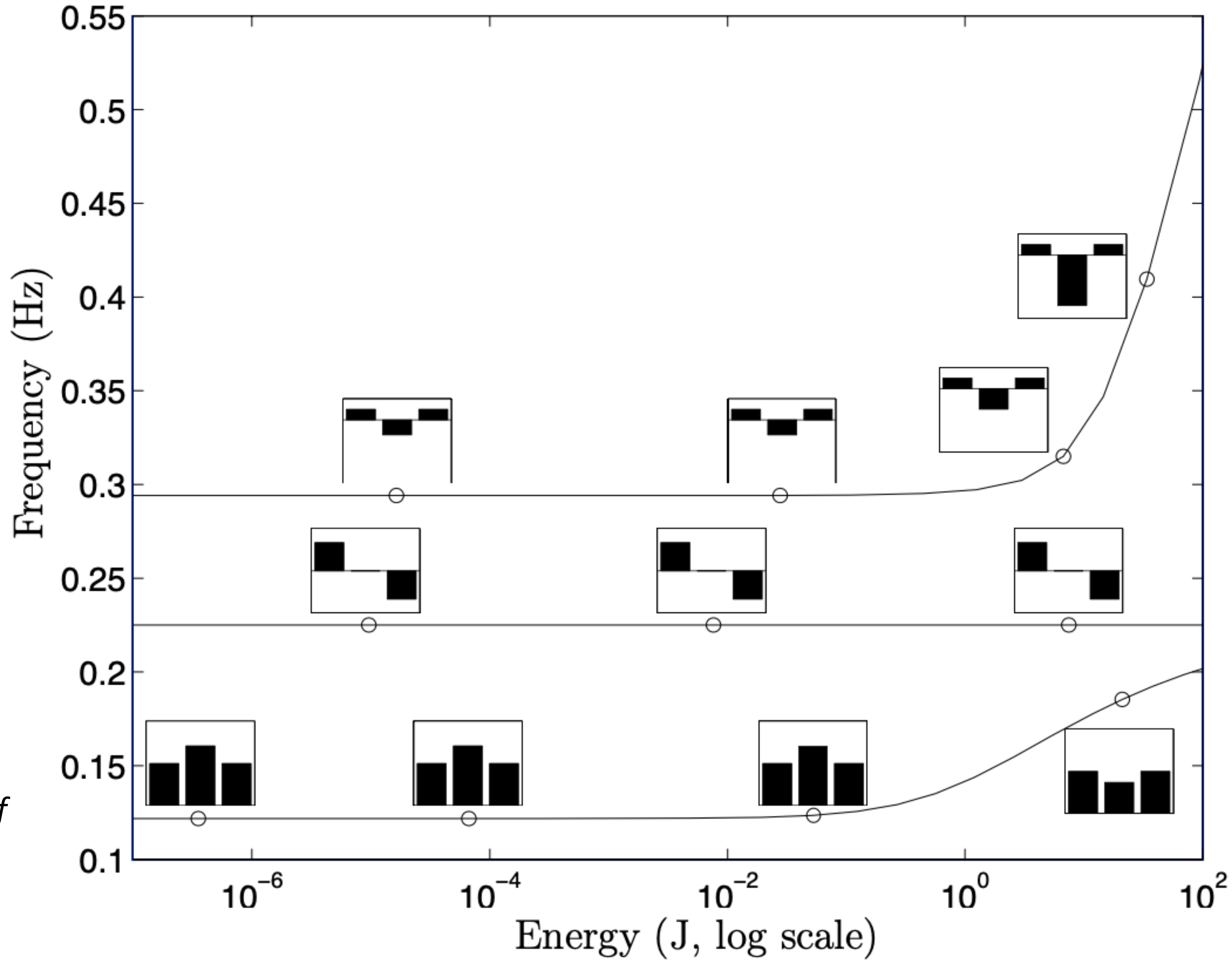
Nonlinear Normal Mode

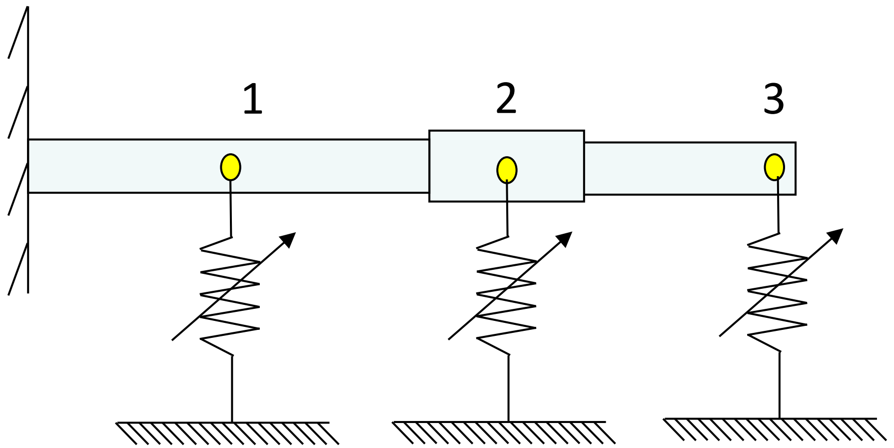
- **NNM** is a function of response level
- At every response level we may have a different mode shape



*Ref: Peeters, M., " Theoretical and Experimental Modal Analysis of Nonlinear Vibrating Structures using Nonlinear Normal Modes", **Ph.D. Thesis**, 2010.*

Response Controlled Sine Test (RCT*)

- However, for a given response level, **mode shape will not change notably** at frequencies around the resonance frequency



- If U_1 is kept constant during stepped-sine test, U_2 and U_3 will also remain constant
- This is great
- We can extend the **Response-Controlled Stepped-Sine Test (RCT*) concept** to systems with many NL elements
- Then we do not even need to know the **number, types and locations of NL elements**

Karaağaçlı, T.; Özgüven, H.N. "Experimental modal analysis of nonlinear systems by using response-controlled stepped-sine testing", **Mechanical Systems and Signal Processing, 2021.*

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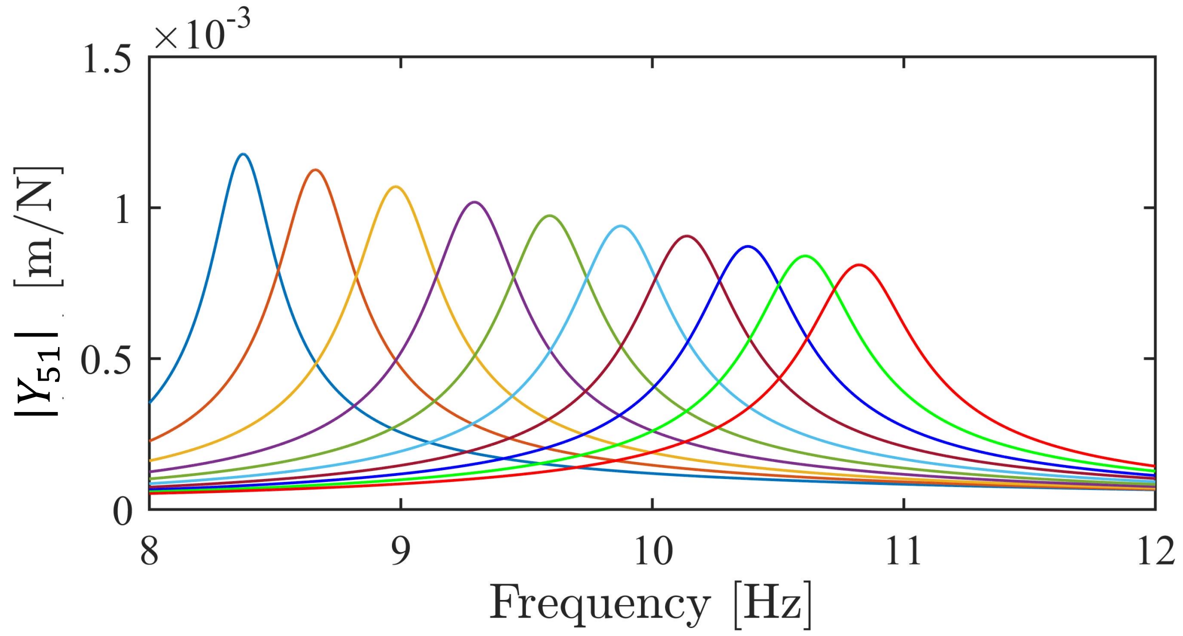
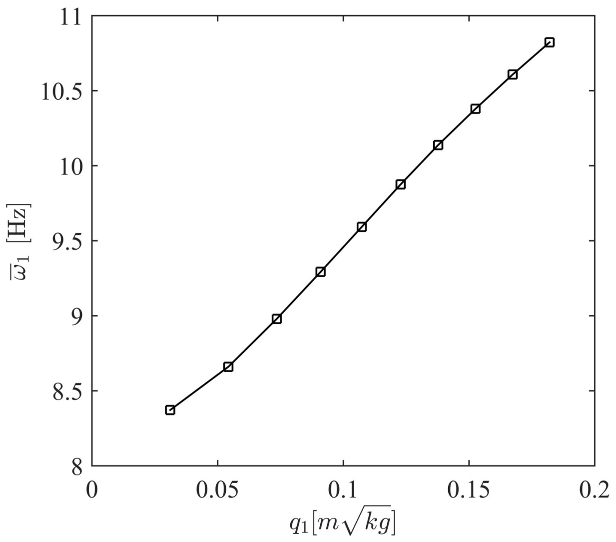
Response Controlled Sine Test (RCT)

- Excite the NL system around a resonance frequency by keeping the amplitude of any point constant
- Obtain quasi-linear FRFs
- Commercially available test systems can be used

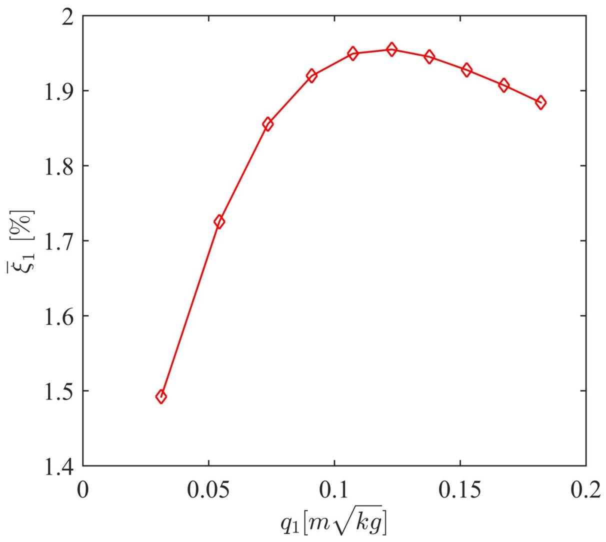


Experimental Nonlinear Modal Analysis

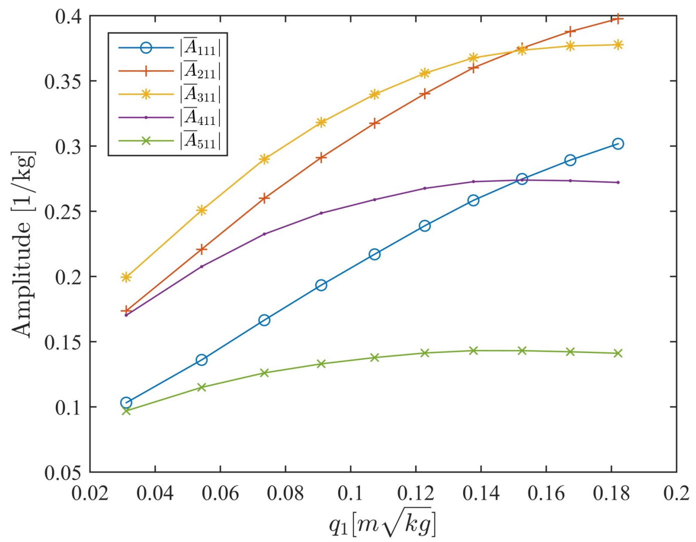
- Identify modal parameters from each quasilinear FRF
- Plot variations of modal parameters as a function of modal amplitude ⇨ Modal model



Modal frequency



Modal damping



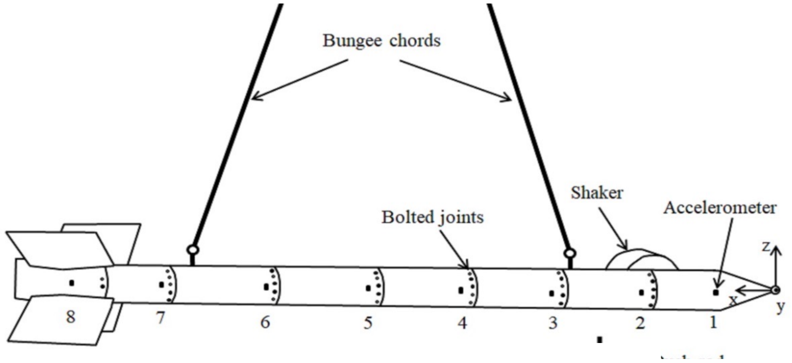
Modal constants

Experimental Nonlinear Modal Analysis

Response to a given harmonic forcing around resonance (in a very similar way to the linear modal analysis):

$$Y_{jk}(\omega, q_r) = \frac{\phi_{jr}(q_r)\, \phi_{kr}(q_r)}{\omega_r^2(q_r) - \omega^2 + i\gamma_r(q_r)\omega_r^2(q_r)}$$

Iteration is required



Identification of individual nonlinear connections (bolts) is not possible

RCT seems to be the best available technique for modelling connection dynamics for such a system

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- Introduction
- Quasi-linear Behaviour of Nonlinear Systems
  - Nonlinearity Matrix Concept
  - RCT – Response Controlled Stepped-Sine Testing
- **Nonlinear Structural Coupling**
- Nonlinear Structural Decoupling and Nonlinear Joint Identification
- Applications

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Nonlinear Structural Coupling

A. By using physical model of the NL sub-structure

B. By using measured quasi-linear FRFs of the NL sub-structure

C. By using modal model of the NL sub-structure identified from measured quasi-linear FRFs

General Comments

- Sub- and super-harmonics are assumed to be negligible
- An iterative solution is required
- Measured or calculated FRFs of the linear sub-structure can be used

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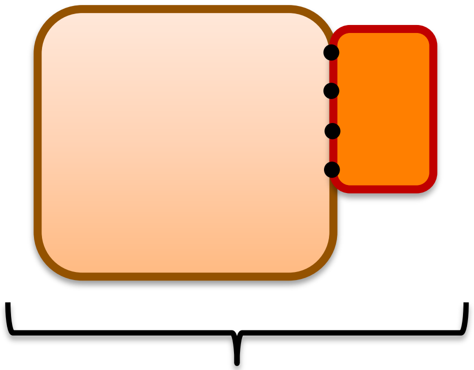
Nonlinear Structural Coupling

A. By using physical model of the NL sub-structure

- A structural coupling method developed for linear systems is extended to nonlinear systems
- In general, FBS methods use either FRF Coupling/Dual Assembly or Impedance Coupling (Primal Assembly).
- In this case, a hybrid approach is used.
- **FRF matrix** of a main structure is coupled with the **dynamic stiffness/impedance matrix** of a smaller sub-structure.

Linear Systems:

Sub-structure A (Main structure): $\boldsymbol{Y}^A$



Sub-structure B: $\boldsymbol{M}^B, \boldsymbol{C}^B, \boldsymbol{H}^B, \boldsymbol{K}^B$

Coupled structure: $\boldsymbol{Y}^{AB}$

ODTÜ METU Structural Coupling by Matrix Inversion Method (MIM)*

Structural coupling: Couple FRFs of a main system with the physical model of a smaller sub-system

Linear Systems:

$$\begin{bmatrix} \boldsymbol{Y}_{ca}^{AB} \\ \boldsymbol{Y}_{ba}^{AB} \end{bmatrix} = \boxed{\left[\begin{bmatrix} \boldsymbol{I} & 0 \\ \boldsymbol{0} & 0 \end{bmatrix} + \begin{bmatrix} \boldsymbol{Y}_{cc}^{A} & 0 \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \boldsymbol{Z}^B \right]^{-1}} \begin{bmatrix} \boldsymbol{Y}_{ca}^{A} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{Y}_{cc}^{AB} & \boldsymbol{Y}_{cb}^{AB} \\ \boldsymbol{Y}_{bc}^{AB} & \boldsymbol{Y}_{bb}^{AB} \end{bmatrix} = \boxed{\left[\begin{bmatrix} \boldsymbol{I} & 0 \\ \boldsymbol{0} & 0 \end{bmatrix} + \begin{bmatrix} \boldsymbol{Y}_{cc}^{A} & 0 \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \boldsymbol{Z}^B \right]^{-1}} \begin{bmatrix} \boldsymbol{Y}_{cc}^{A} & 0 \\ 0 & \boldsymbol{I} \end{bmatrix}$$

$$\boldsymbol{Y}_{aa}^{AB} = \boldsymbol{Y}_{aa}^{A} - (\boldsymbol{Y}_{ac}^{A}|0)\boldsymbol{Z}^B \begin{bmatrix} \boldsymbol{Y}_{ca}^{AB} \\ \boldsymbol{Y}_{ba}^{AB} \end{bmatrix}$$

$$(\boldsymbol{Y}_{ac}^{AB}|\boldsymbol{Y}_{ab}^{AB}) = (\boldsymbol{Y}_{a,c}^{A}|0)\left[\boldsymbol{I} - \boldsymbol{Z}^B \begin{bmatrix} \boldsymbol{Y}_{cc}^{AB} & \boldsymbol{Y}_{cb}^{AB} \\ \boldsymbol{Y}_{bc}^{AB} & \boldsymbol{Y}_{bb}^{AB} \end{bmatrix}\right]$$

a: coordinates of original system (A) only

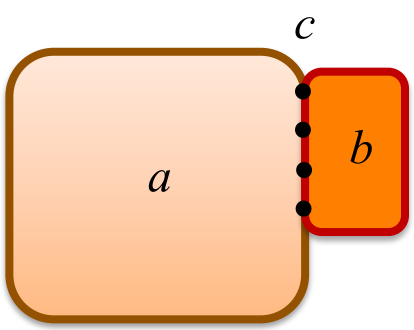
c: common coordinates

b: coordinates of coupled subsystem (B) only

m: coordinates of B (b+c)

Order of the matrix to be inverted: $n_m = n_b + n_c$

No matrix inversion



*Özgüven, H. N., "Structural Modifications Using Frequency Response Functions", ***Mechanical Systems and Signal Processing***, 1990.*

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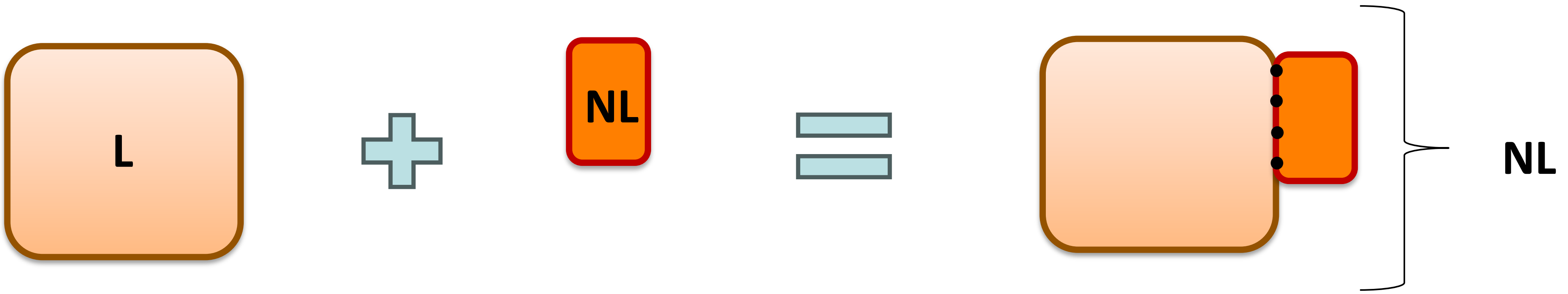
Nonlinear Structural Coupling*

A. By using **physical model** of the NL sub-structure

Sub-structure A (Main system) | Sub-structure B | Coupled structure



$\boldsymbol{Y^A} + \boldsymbol{M^B, C^B, H^B, K^B}, \boxed{\Delta(\boldsymbol{U_m})} \rightarrow \boldsymbol{Y^{AB}}$

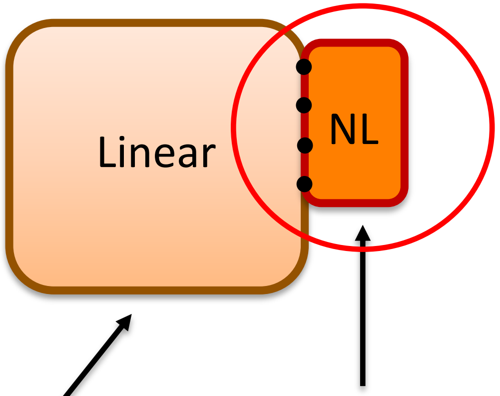
FRF of the system (Analytical or Experimental)

System matrices (Analytical)

FRF of the coupled system (MIM)

*Kalaycıoğlu, T., Özgüven, H. N., "Nonlinear Structural Modification and Nonlinear Coupling", ***Mechanical Systems and Signal Processing,*** 2014

Nonlinear Structural Coupling



Main Structure
Sub-structure **A**

Coupled Structure
Sub-structure **B**

$$\underbrace{\left(-\omega^2 \boldsymbol{M}^B + i\omega \boldsymbol{C}^B + i\boldsymbol{H}^B + \boldsymbol{K}^B\right)}_{\boldsymbol{Z}^L} \boldsymbol{U}_m + \boxed{\Delta(\boldsymbol{U}_m)\, \boldsymbol{U}_m} = \boldsymbol{F}$$

Dynamic stiffness matrix of the linear part of sub-structure B

$\boldsymbol{Z}^B$: ***Dynamic stiffness matrix*** of the nonlinear sub-structure B

$$\boxed{\boldsymbol{Z}^B = \boldsymbol{Z}^L + \Delta(\boldsymbol{U}_m)}$$

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Nonlinear Structural Coupling

Nonlinear Systems:

$$\begin{bmatrix} \boldsymbol{Y}^{AB}_{c,a} \\ \boldsymbol{Y}^{AB}_{b,a} \end{bmatrix} = \left[\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \boldsymbol{Y}^{A}_{c,c} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \boldsymbol{Z}^B \right]^{-1} \begin{bmatrix} \boldsymbol{Y}^{A}_{c,a} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{Y}^{AB}_{c,c} & \boldsymbol{Y}^{AB}_{c,b} \\ \boldsymbol{Y}^{AB}_{b,c} & \boldsymbol{Y}^{AB}_{b,b} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}^{A}_{c,c} & 0 \\ 0 & \boldsymbol{I} \end{bmatrix} \left[\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \boldsymbol{Y}^{A}_{c,c} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \boldsymbol{Z}^B \right]^{-1}$$

$$\boldsymbol{Y}^{AB}_{a,a} = \boldsymbol{Y}^{A}_{a,a} - \left(\boldsymbol{Y}^{A}_{a,c} \middle| 0 \right) \boldsymbol{Z}^B \begin{bmatrix} \boldsymbol{Y}^{AB}_{c,a} \\ \boldsymbol{Y}^{AB}_{b,a} \end{bmatrix}$$

$$\left(\boldsymbol{Y}^{AB}_{a,c} \middle| \boldsymbol{Y}^{AB}_{a,b} \right) = \left(\boldsymbol{Y}^{A}_{a,c} \middle| 0 \right) \left[\boldsymbol{I} - \boldsymbol{Z}^B \begin{bmatrix} \boldsymbol{Y}^{AB}_{c,c} & \boldsymbol{Y}^{AB}_{c,b} \\ \boldsymbol{Y}^{AB}_{b,c} & \boldsymbol{Y}^{AB}_{b,b} \end{bmatrix} \right]$$

1

$$\boldsymbol{Z}^B = \boldsymbol{Z}^L + \Delta(\boldsymbol{U}_{\boldsymbol{m}})$$

Iteration loop

3

$$\boldsymbol{U}_{\boldsymbol{m}} = \boldsymbol{Y}^{AB}\ \boldsymbol{F}$$

2

Nonlinear Structural Coupling

$$\begin{bmatrix} \boldsymbol{Y}^{AB}_{c,a} \\ \boldsymbol{Y}^{AB}_{b,a} \end{bmatrix} = \left[\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \boldsymbol{Y}^{A}_{c,c} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \boldsymbol{Z}^B \right]^{-1} \begin{bmatrix} \boldsymbol{Y}^{A}_{c,a} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{Y}^{AB}_{c,c} & \boldsymbol{Y}^{AB}_{c,b} \\ \boldsymbol{Y}^{AB}_{b,c} & \boldsymbol{Y}^{AB}_{b,b} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}^{A}_{c,c} & 0 \\ 0 & \boldsymbol{I} \end{bmatrix} \left[\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \boldsymbol{Y}^{A}_{c,c} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \boldsymbol{Z}^B \right]^{-1}$$

$$\boldsymbol{Y}^{AB}_{a,a} = \boldsymbol{Y}^{A}_{a,a} - \left(\boldsymbol{Y}^{A}_{a,c} \middle| 0\right) \boldsymbol{Z}^B \begin{bmatrix} \boldsymbol{Y}^{AB}_{c,a} \\ \boldsymbol{Y}^{AB}_{b,a} \end{bmatrix}$$

$$\left(\boldsymbol{Y}^{AB}_{a,c} \middle| \boldsymbol{Y}^{AB}_{a,b}\right) = \left(\boldsymbol{Y}^{A}_{a,c} \middle| 0\right) \left[\boldsymbol{I} - \boldsymbol{Z}^B \begin{bmatrix} \boldsymbol{Y}^{AB}_{c,c} & \boldsymbol{Y}^{AB}_{c,b} \\ \boldsymbol{Y}^{AB}_{b,c} & \boldsymbol{Y}^{AB}_{b,b} \end{bmatrix} \right]$$

1

$$\boldsymbol{Z}^B = \boldsymbol{Z}^L + \Delta(\boldsymbol{U_m})$$

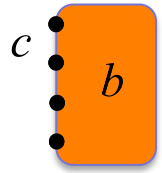
Iteration loop

3

$$\boldsymbol{U_m} = \boldsymbol{Y}^{AB}\ \boldsymbol{F}$$

The related parts of FRFs

2



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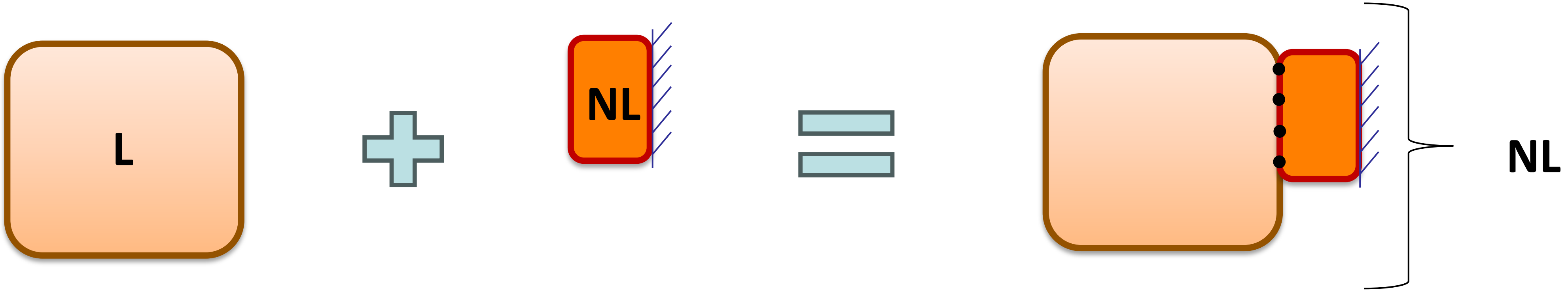
Nonlinear Structural Coupling

B. By using measured quasi-linear FRFs of the NL sub-structure – via RCT

Sub-structure A (Main system) | Sub-structure B | Coupled structure



$$\boldsymbol{Y}^A \quad + \quad \boldsymbol{Y}^B(q_r) \quad \rightarrow \quad \boldsymbol{Y}^{AB}(q_r)$$

FRF of the system

(Analytical or Experimental)

Quasi-linear FRFs of the system

(Experimental - RCT)

Quasi-linear FRFs of the coupled system

(via FRF coupling)

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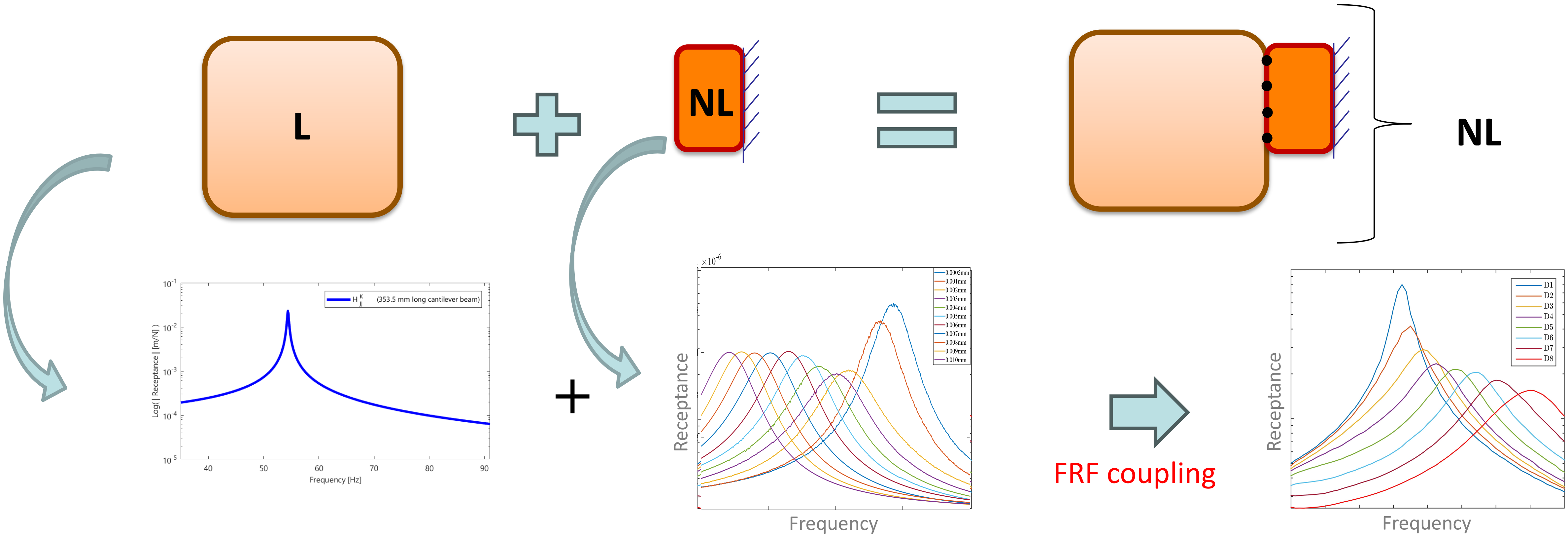
Nonlinear Structural Coupling



Measured/calculated FRF

Measured quasi-linear FRFs

Quasi-linear FRFs of the assembly

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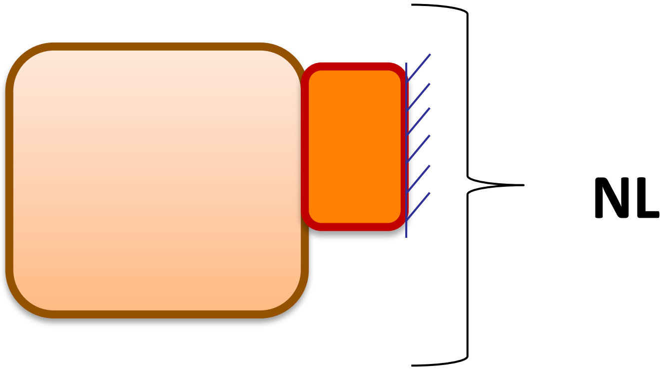
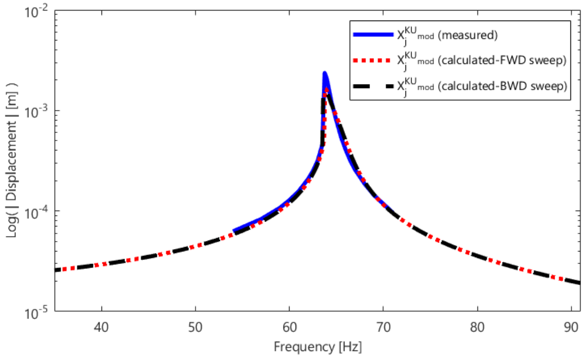
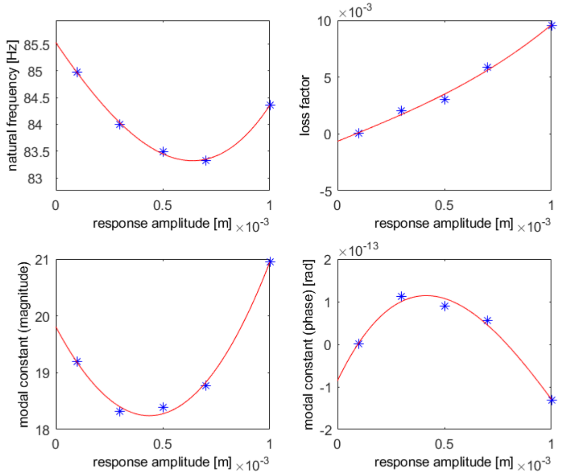
Nonlinear Structural Coupling





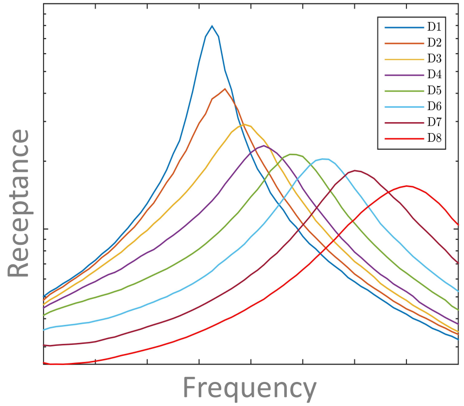
Frequency response of the coupled system for a given forcing

Iterative computation



Response dependent modal parameters

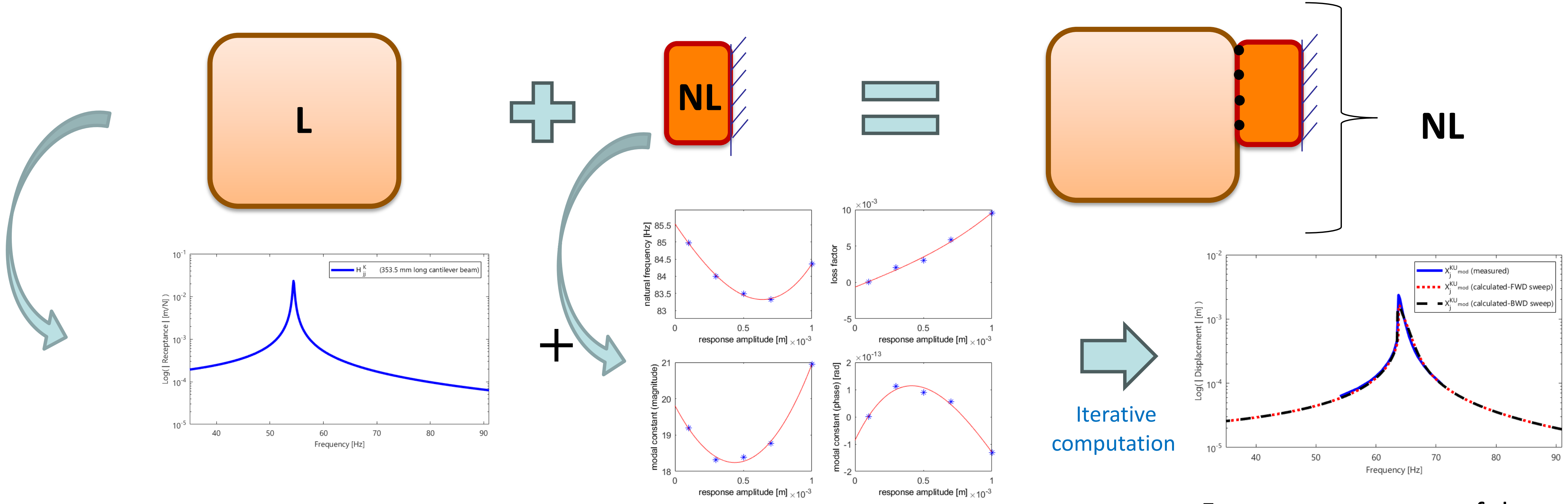
Modal identification



Calculate quasi-linear FRFs of the assembly

Nonlinear Structural Coupling*

C. By using modal model of the NL sub-structure identified from measured quasi-linear FRFs



Measured/calculated FRF

Identified NL modal model

Frequency response of the coupled system for a given forcing

*Arslan, Ö. and Özgüven, H. N., "Modal Identification of Non-Linear Structures and the Use of Modal Model in Structural Dynamic Analysis", **IMAC 26**, Orlando, Florida, February 4-7, 2008.

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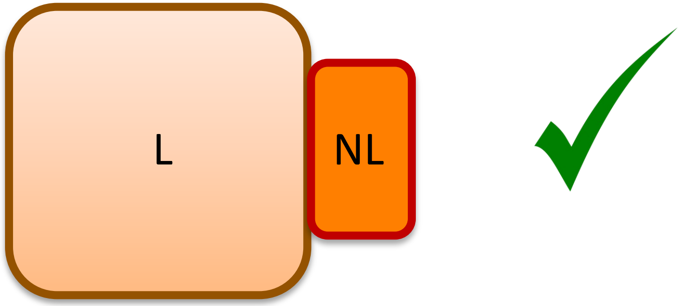
Nonlinear Structural Coupling

- Until now we considered a small NL sub-structure coupled to a L main structure



- What if we have just the opposite: a smaller L sub-structure coupled to a large NL sub-structure?

A typical application

NL system: aircraft-pylon-store assembly
L system: additional mass to represent a heavier store

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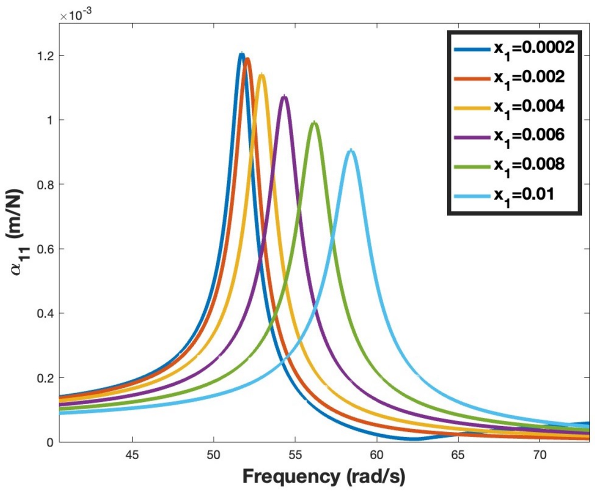
Nonlinear Structural Coupling

A. By using **physical model** of L sub-system*

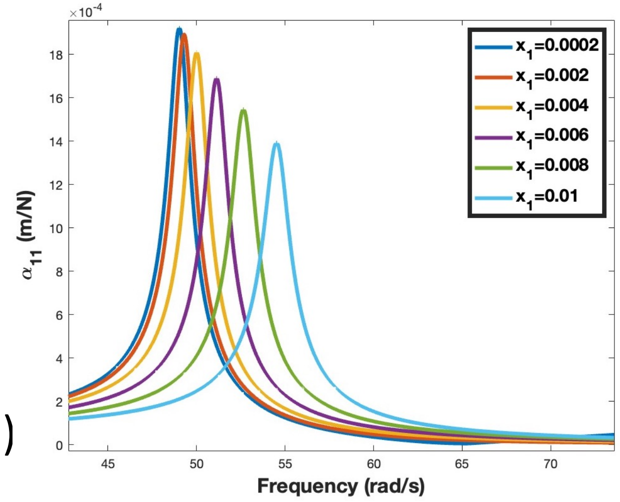


Measured quasi-linear FRFs

$+$

$$\boldsymbol{M^B}, \boldsymbol{C^B}, \boldsymbol{H^B}, \boldsymbol{K^B}$$

Structural Coupling
(no iteration at this stage)



Calculate quasi-linear FRFs of the assembly

*Ekinci, Özer, Özgüven, *"A Novel Approach for Local Structural Modification of Nonlinear Structures"*, ***IMAC 41***, *2023*

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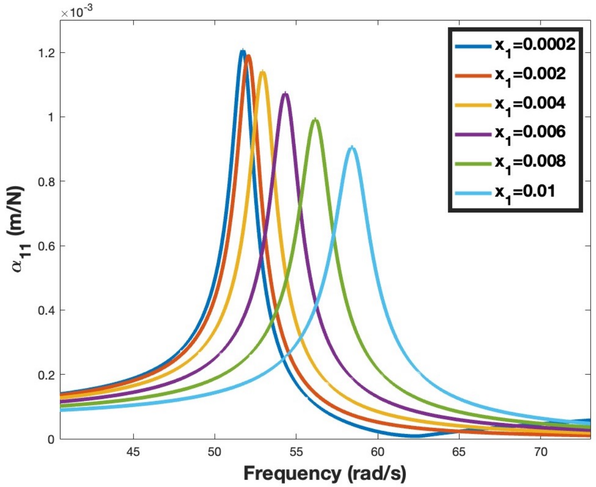
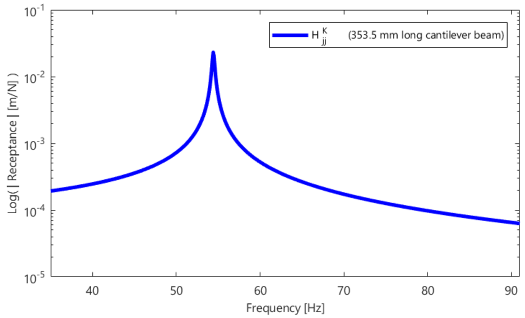
Nonlinear Structural Coupling

B. By using FRF of L sub-system



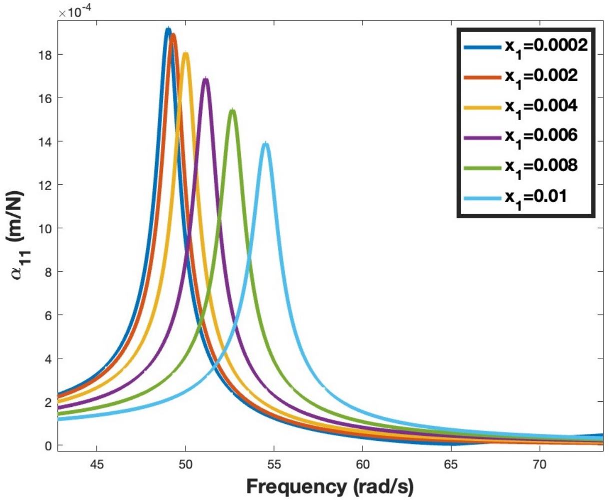
Measured quasi-linear FRFs

+



Measured/calculated FRF

FRF coupling (no iteration at this stage)



Calculate quasi-linear FRFs of the assembly

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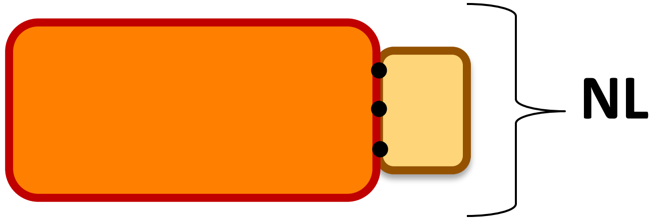
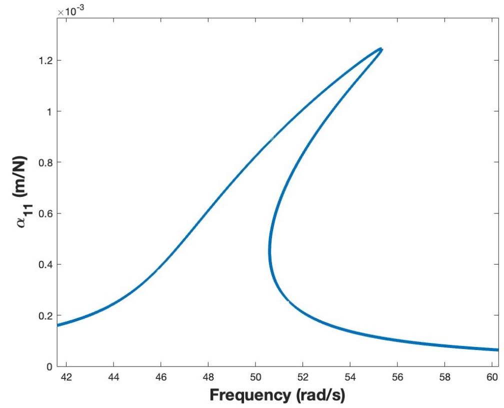
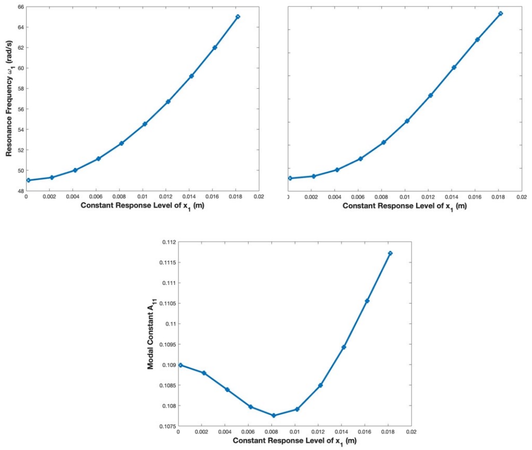
Nonlinear Structural Coupling





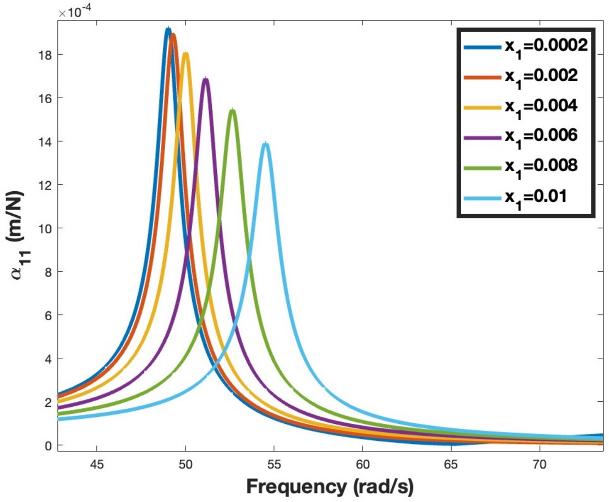
Frequency response of the coupled system for a given forcing level

Iterative computation



Response dependent modal parameters

Modal identification



Quasi-linear FRFs of the assembly

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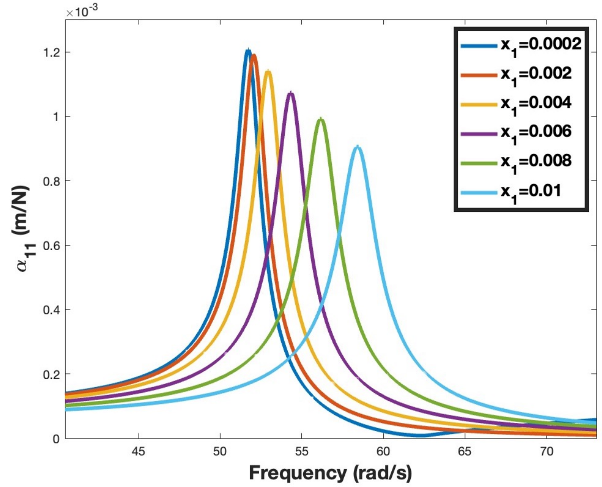
Nonlinear Structural Coupling

Using quasi-linear FRFs of the NL sub-structure or the modal model identified from quasi-linear FRFs

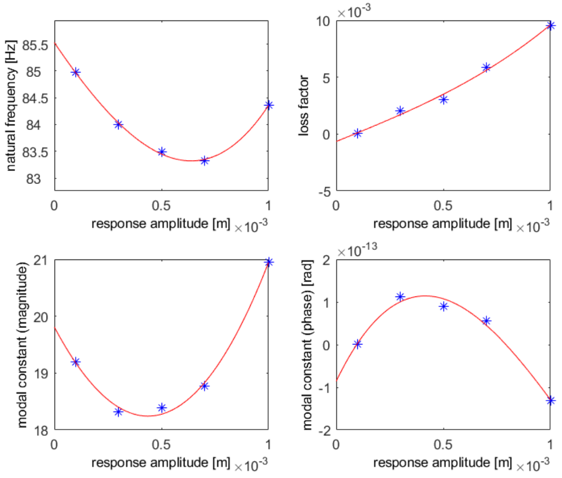
- Simple in theory (with some assumptions)
- May not be so simple in real applications
  - Measurement difficulties
  - Limitations
- Good results in several applications

- Advantages:
  - No need to know the **number of NL elements**
  - No need to know the **location of NL elements**
  - No need to know the **types of NLs**
  - No need to **identify individual NL elements**



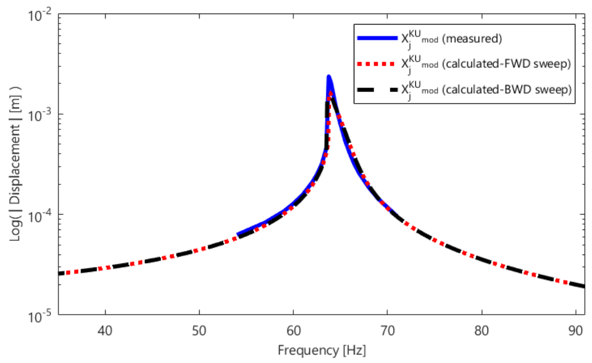
Measured quasi-linear FRFs

or



Identified NL modal model

Iterative computation



Frequency response of the coupled system for a given forcing

ODTÜ
METU

- Introduction
- Quasi-linear Behaviour of Nonlinear Systems
  - Nonlinearity Matrix Concept
  - RCT – Response Controlled Stepped-Sine Testing
- Nonlinear Structural Coupling
- **Nonlinear Structural Decoupling and Nonlinear Joint Identification**
- Applications

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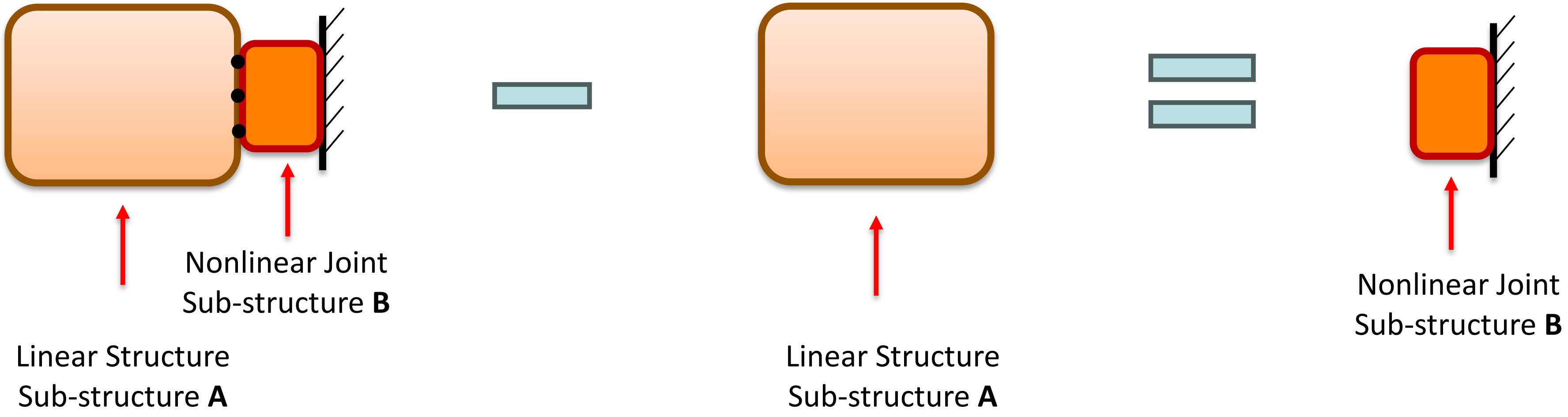
METU

Nonlinear Structural Decoupling

A simple case



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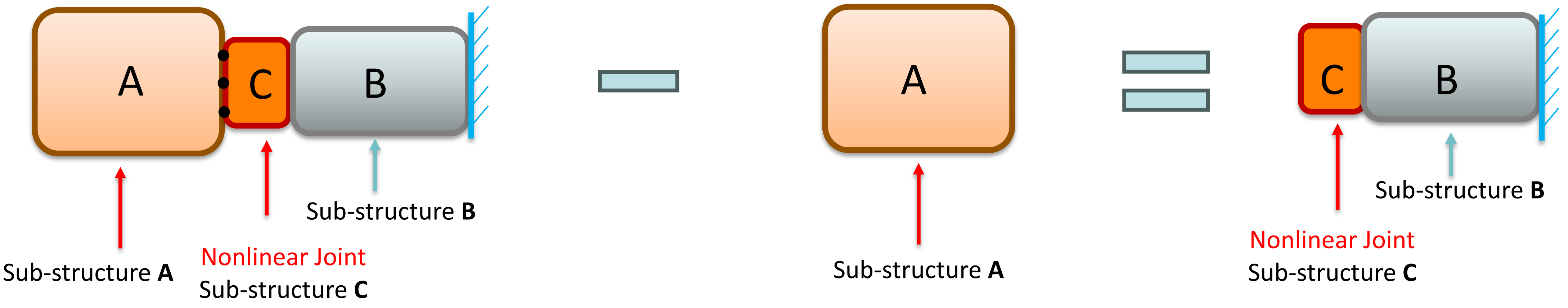
Nonlinear Structural Decoupling

FRF Decoupling Method for Nonlinear Systems (FDM-NS)*



Coupled System

Uncoupled System

*Kalaycıoğlu, T., Özgüven, H. N., *"FRF Decoupling of nonlinear systems", **Mechanical Systems and Signal Processing**, 2018.*

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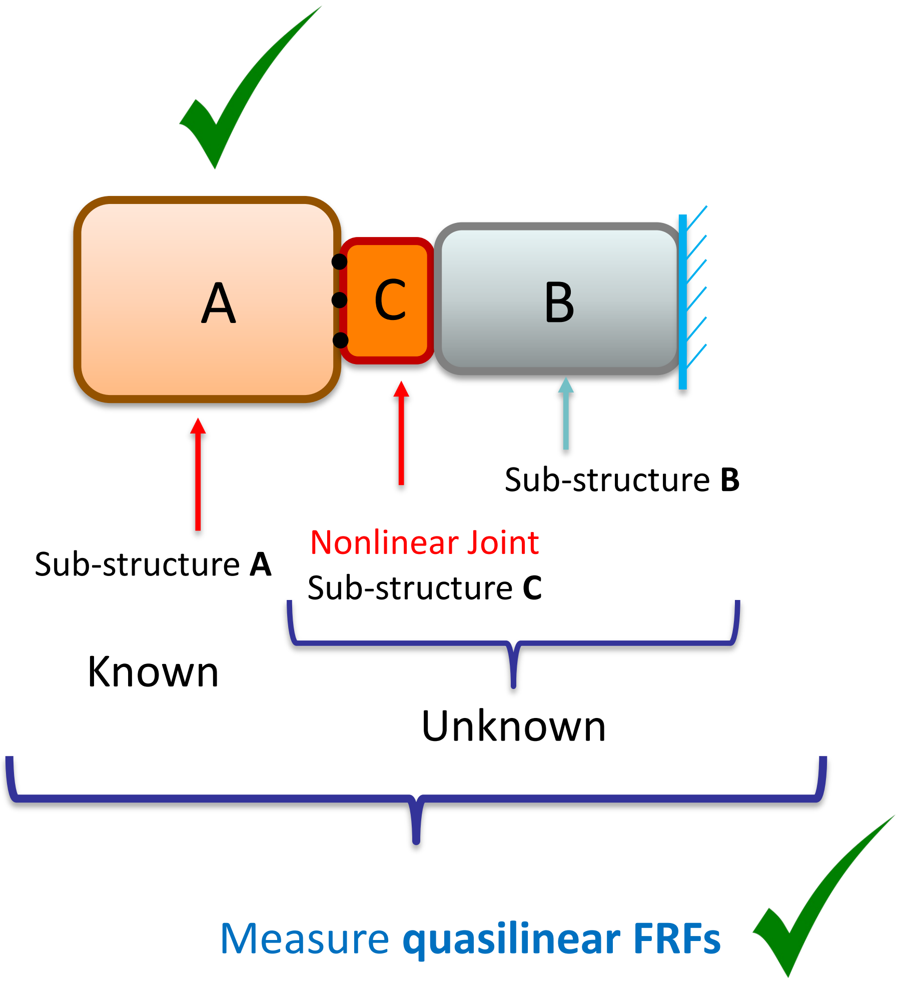
Nonlinear Structural Decoupling



Step 1

Measure **quasi-linear FRFs** of the Coupled System (**A** + **B** + **C**) using RCT at various displacement levels

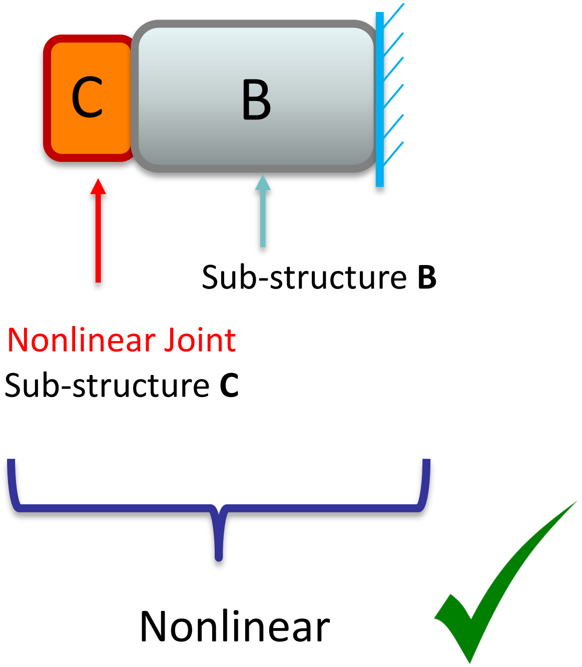
Step 2

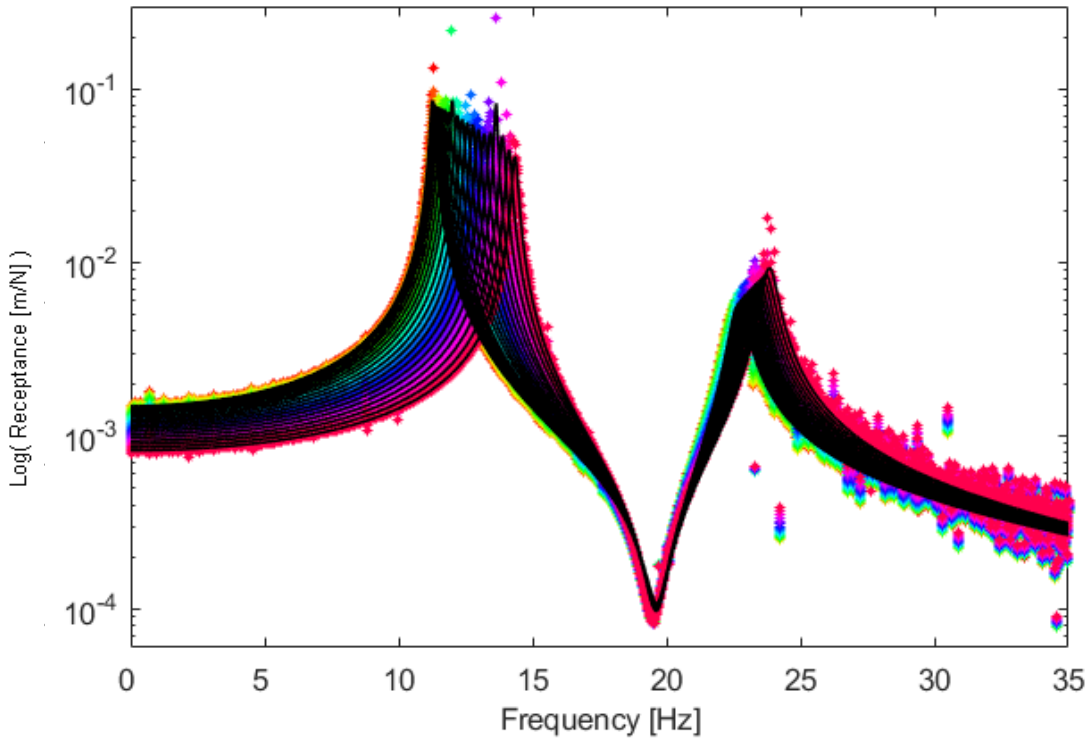
Calculate (or measure) FRFs of **A** for the DOFs being interested in + connection DOFs

Nonlinear Structural Decoupling

Step 3

Apply a frequency based decoupling technique for linear systems* to obtain ⇒ **quasi-linear FRFs** of **(B + C)** for various response levels



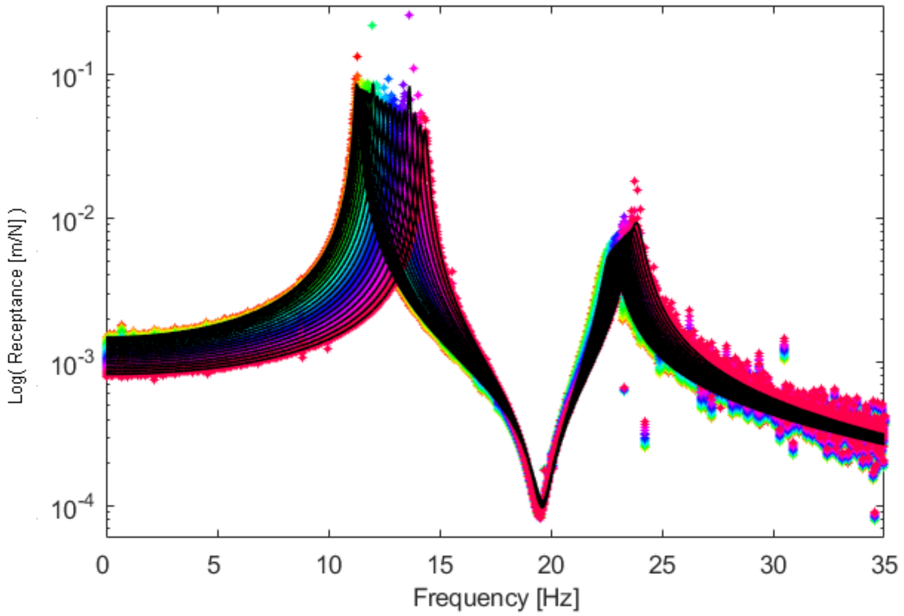


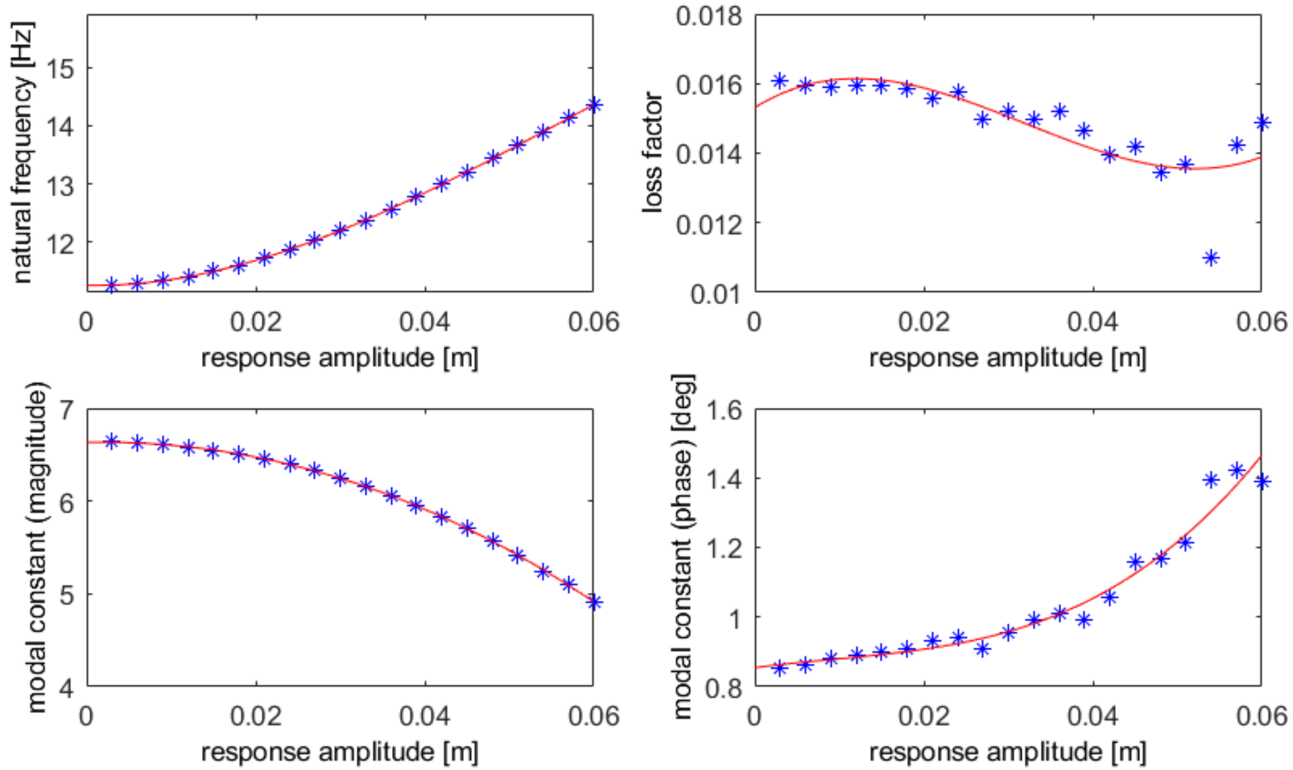
*W. D'Ambrogio, A. Fregolent, ¨The role of interface DOFs in decoupling of substructures based on the dual domain decomposition¨, ***Mechanical Systems and Signal Processing.***, 2010

Nonlinear Structural Decoupling

Step 4

Identify modal parameters of the unknown nonlinear system (**B** + **C**) from each quasi-linear FRF





First Mode

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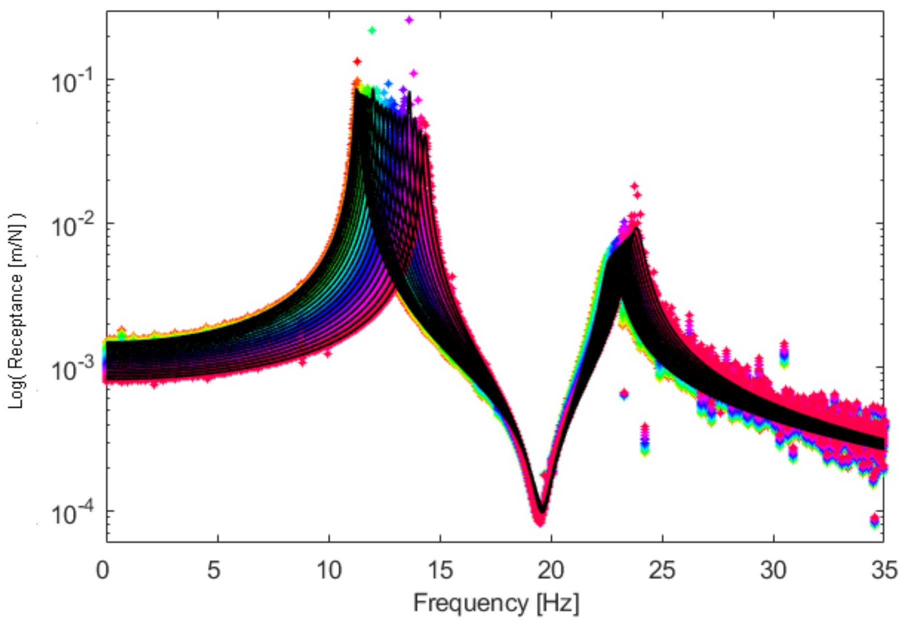
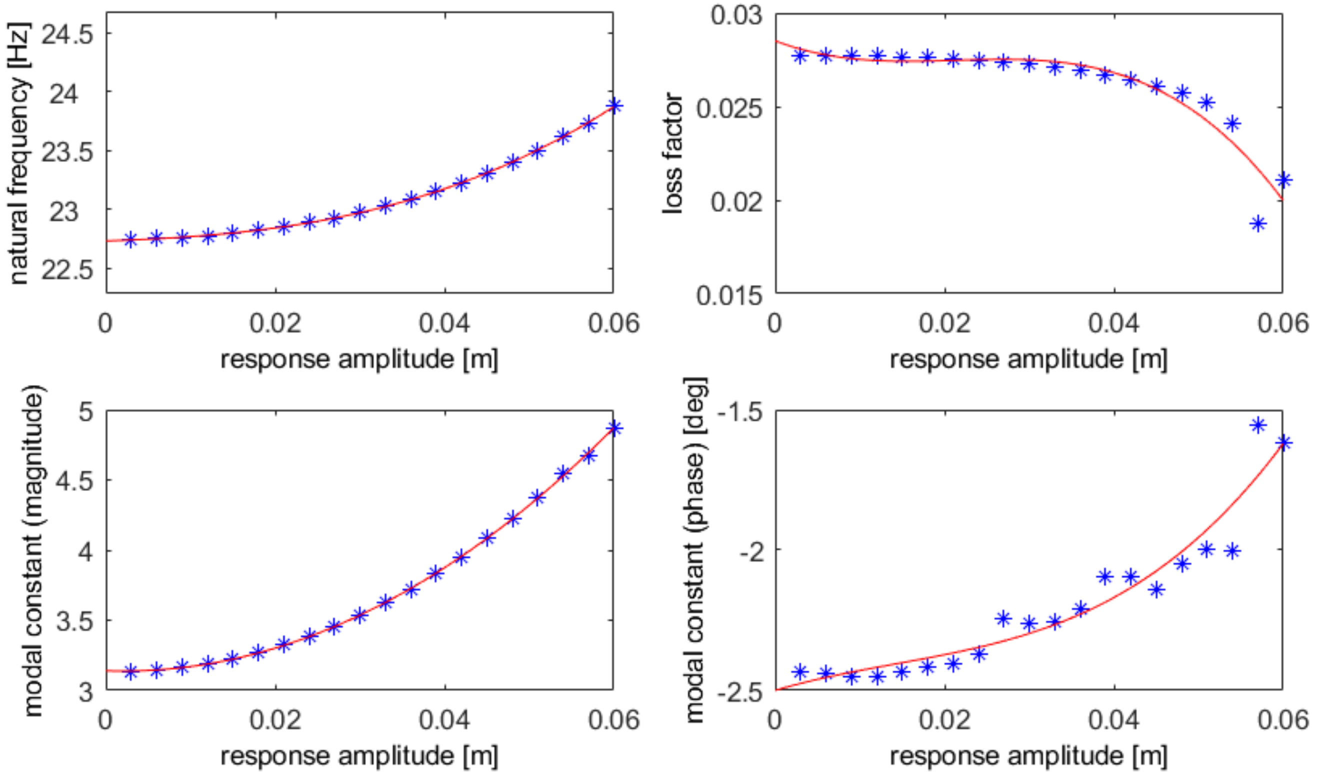
Nonlinear Structural Decoupling

Step 4

Identify modal parameters of the unknown nonlinear system (**B** + **C**) from each quasi-linear FRF





Second Mode

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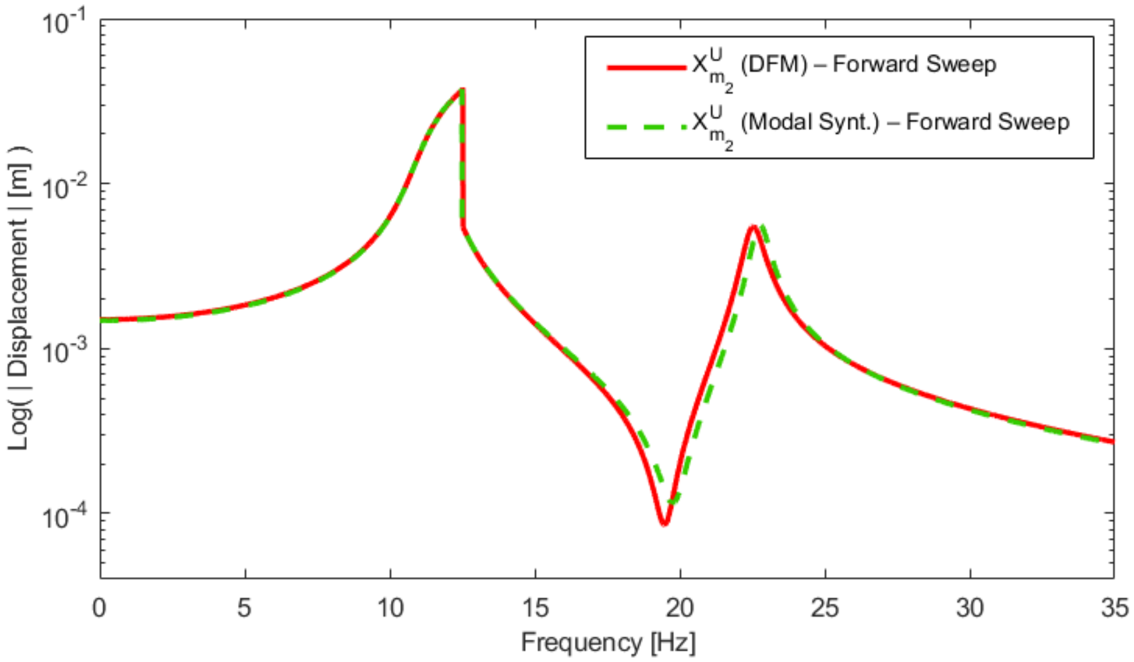
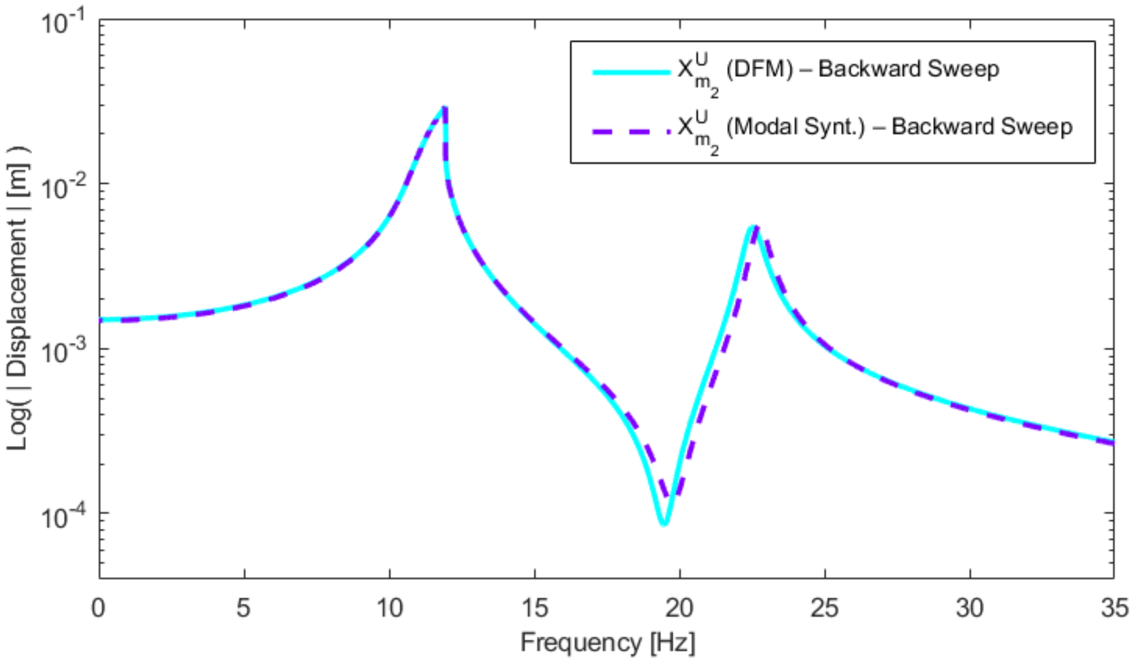
Nonlinear Structural Decoupling

Step 5

Calculate the frequency response of the unknown nonlinear system from identified modal parameters – iterative computation





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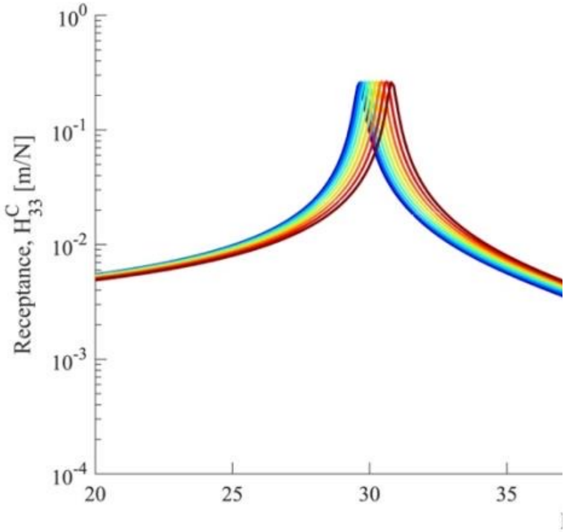
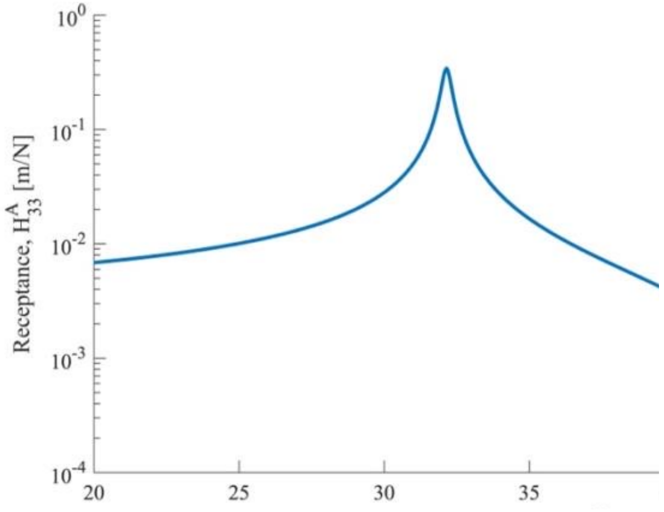
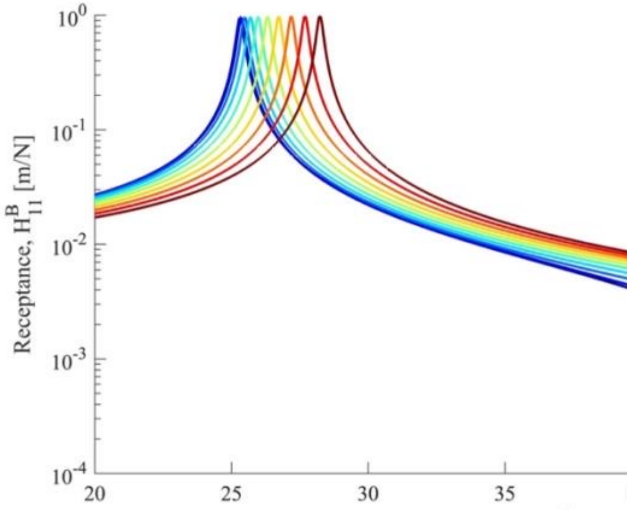
Nonlinear Structural Decoupling



Measured quasilinear FRFs of the coupled system



Measured/calculated FRFs of sub-structure A



Calculated quasilinear FRFs of sub-structure B

- Simple in theory
- May not be so simple in real applications due to
  - Limitations
  - Measurement difficulties

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- Introduction
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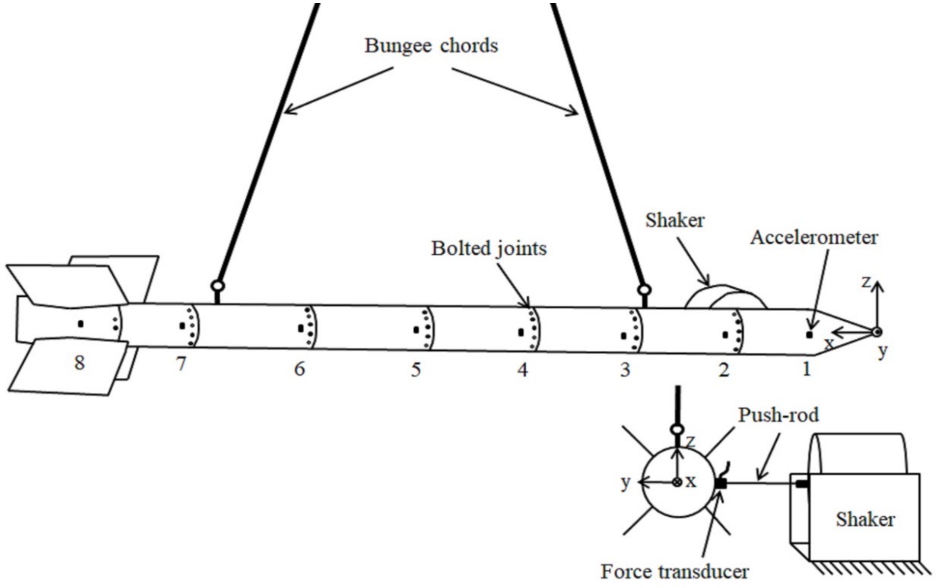
- Nonlinear Structural Decoupling and Nonlinear Joint Identification
- **Applications**

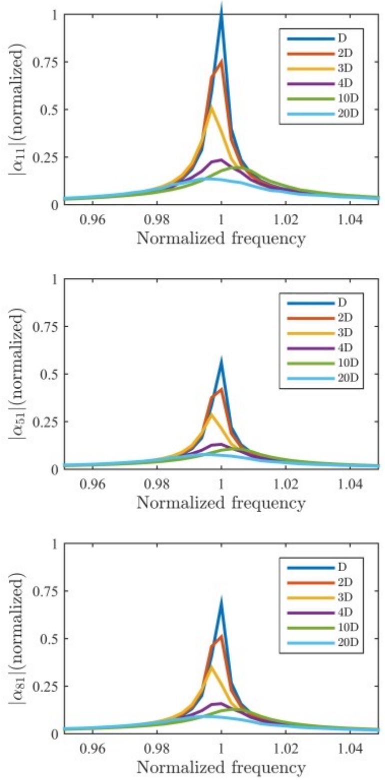
ODTÜ METU

Application – 1

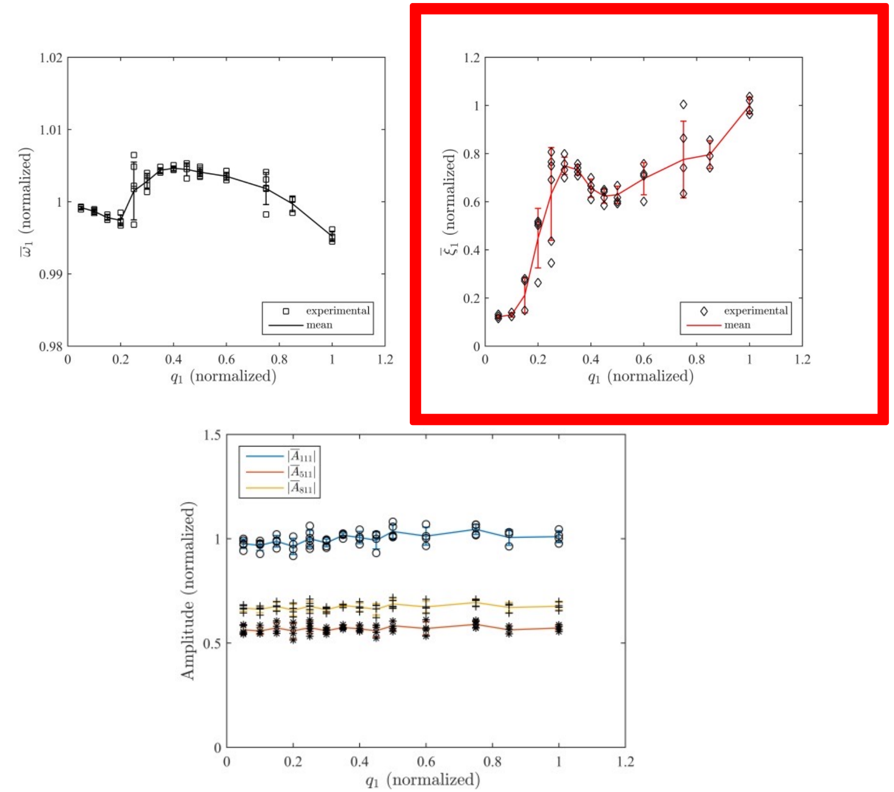
Real Missile Structure*



Measurement of FRFs by RCT



Identification of nonlinear modal parameters



Bolted connections ⇨ high damping NL

*Karaağaçlı, T.; Özgüven, H.N. "Experimental modal analysis of nonlinear systems by using response-controlled stepped-sine testing", **Mechanical Systems and Signal Processing**, v. 146, 2021.

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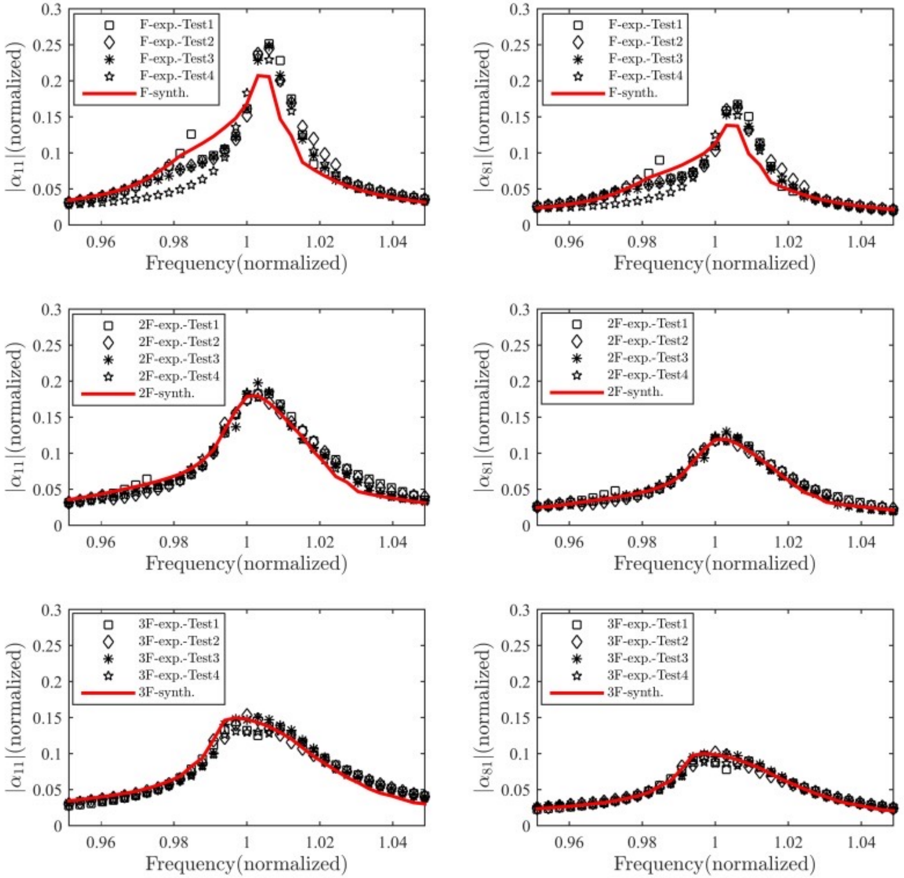
Application – 1

Real Missile Structure

- Constant force amplitude FRFs are calculated by using identified nonlinear modal parameters.

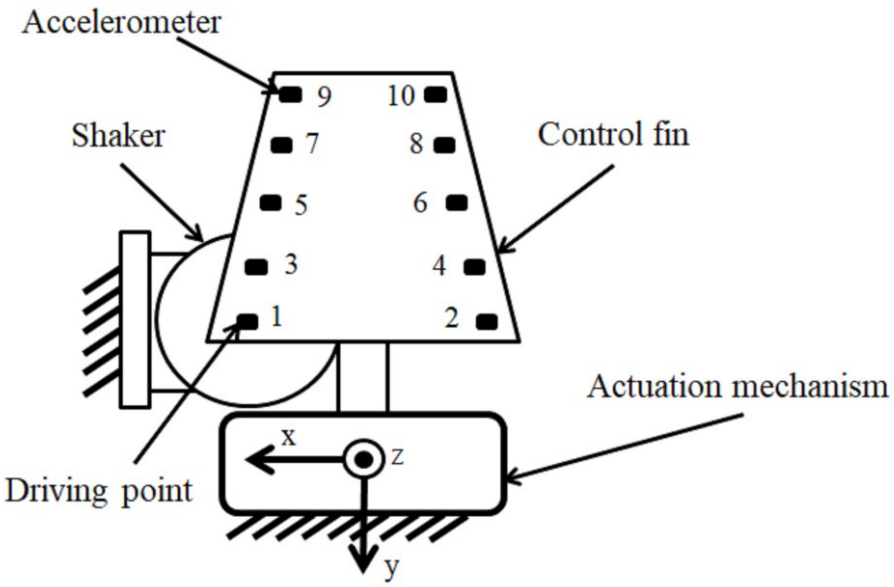
$$\alpha_{jk}(\omega, q_r) = \frac{A^r_{jk}(q_r)}{\bar{\omega}_r^2(q_r) - \omega^2 + i2\bar{\xi}_r(q_r)\omega\bar{\omega}_r(q_r)}$$

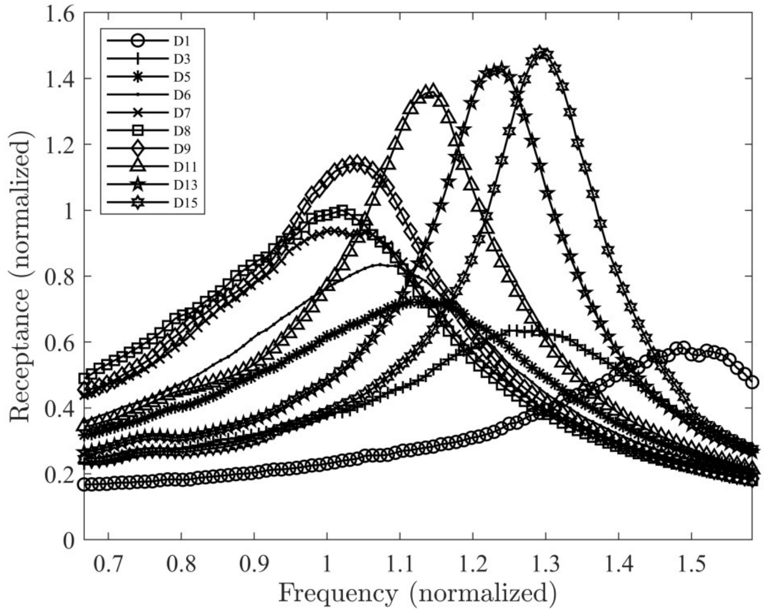
- Calculated FRFs agree well with measured constant force FRFs

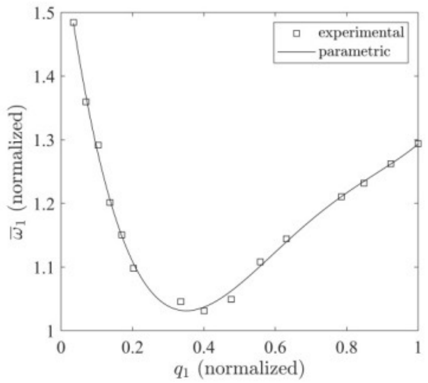


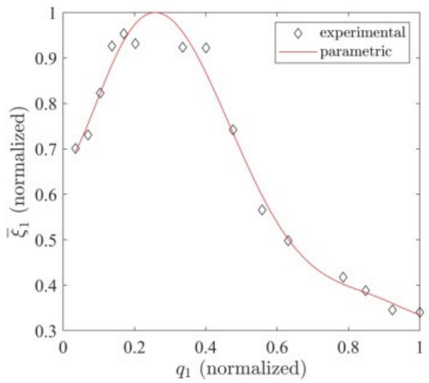
Application – 2

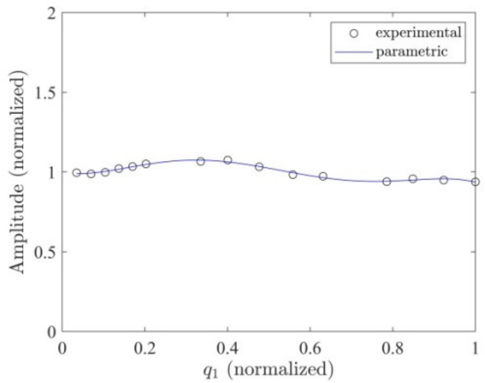
Control Fin – Actuation Mechanism*











NL due to backlash and friction
(very high stiffness and damping NL)

*Karaağaçlı, T.; Özgüven, H.N. *"Experimental Modal Analysis of Structures with High Nonlinear Damping by using RCT"*, ***IMAC 41***, *2023.*

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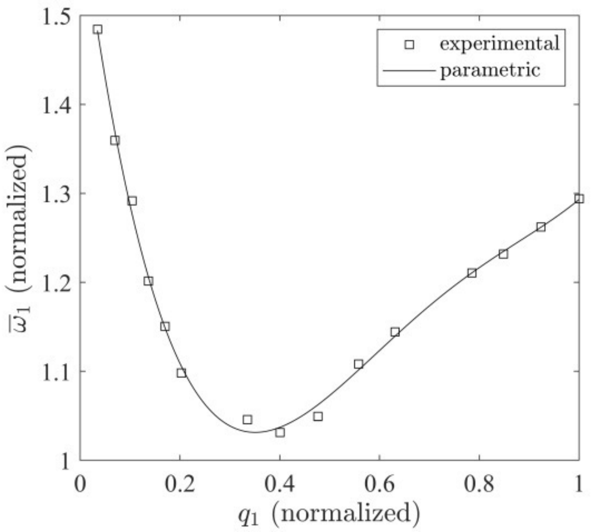
Application – 2

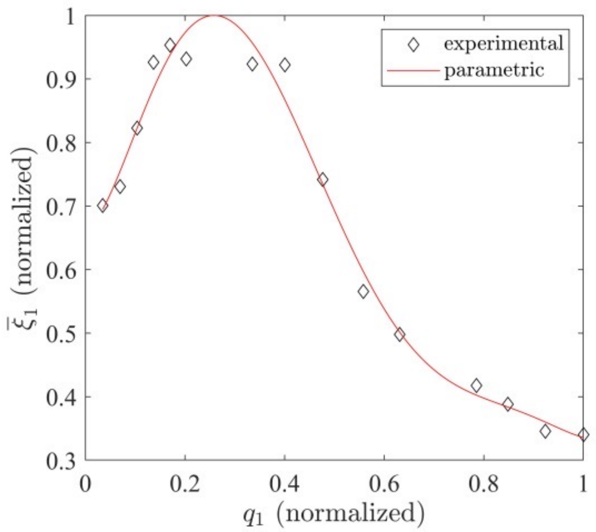
Control Fin – Actuation Mechanism

Natural Frequency



~ 50% change

Modal Damping Ratio

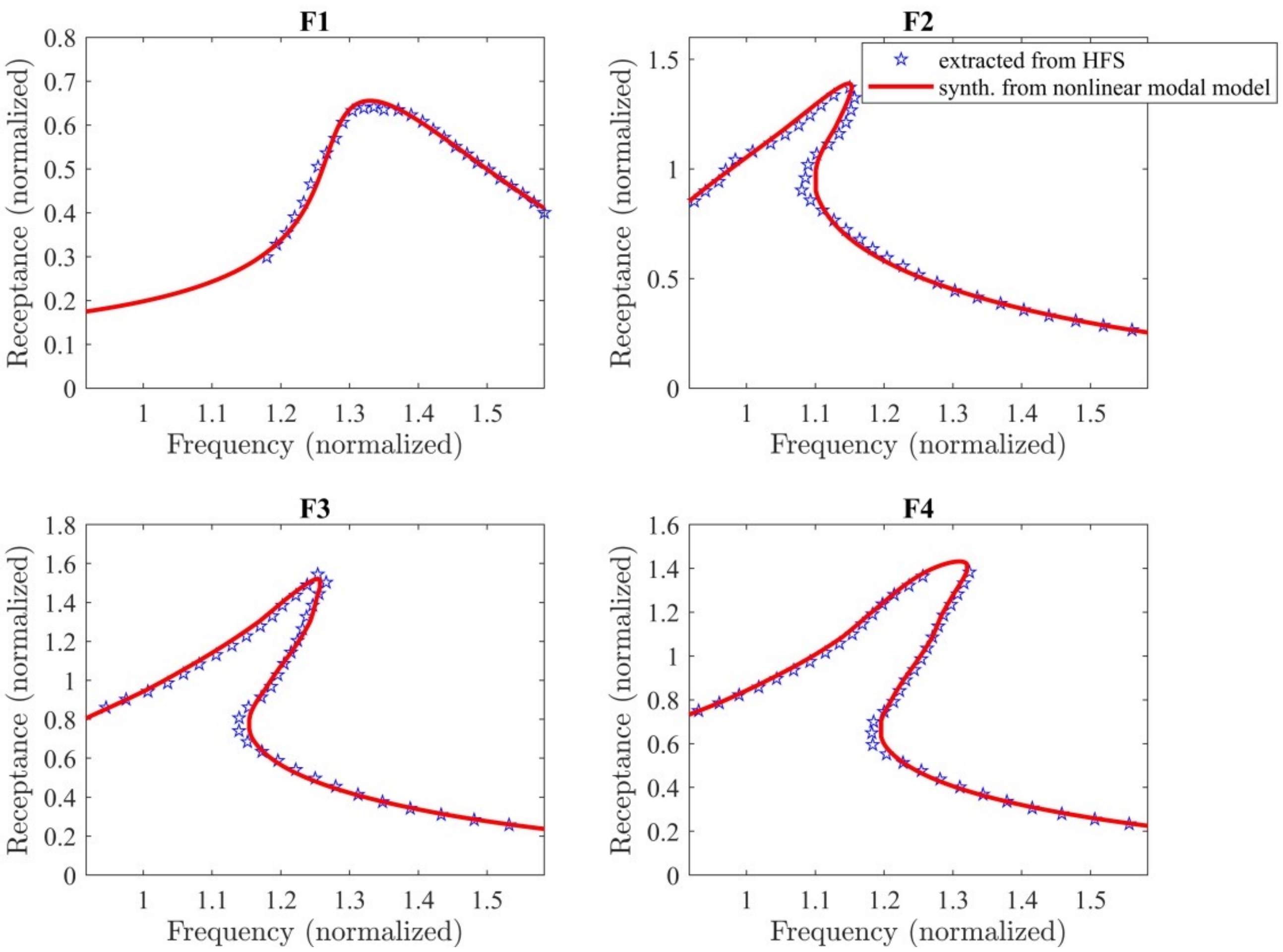


~ 200% change

- A very high degree of stiffness and damping NL that cannot be reached in many studies
- In real values, damping goes up to 15%

Applications – 2

Control Fin – Actuation Mechanism



Constant-force FRFs

Synthesized: From modal model

HFS: Purely experimental (but from RCT)

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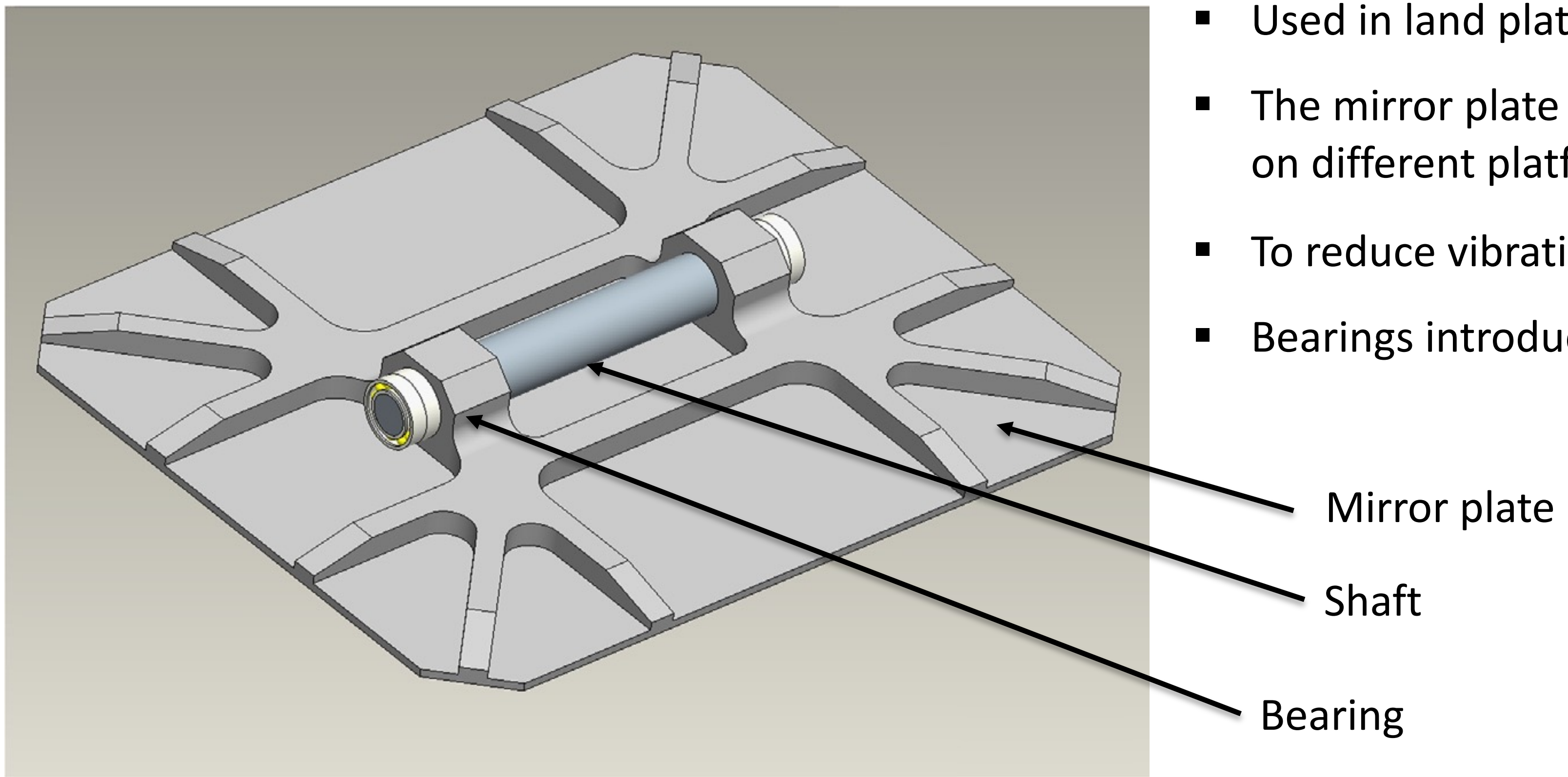
Applications – 3

Mirror plate-shaft assembly*

- Used in land platforms for optical purposes
- The mirror plate is a costly part – it is designed and used on different platforms
- To reduce vibration, shaft-bearing design is modified
- Bearings introduce cubic stiffness



*Kalaycıoğlu, T., Özgüven, H. N., "Nonlinear Structural Modification and Nonlinear Coupling", ***Mechanical Systems and Signal Processing***, 2014 .*

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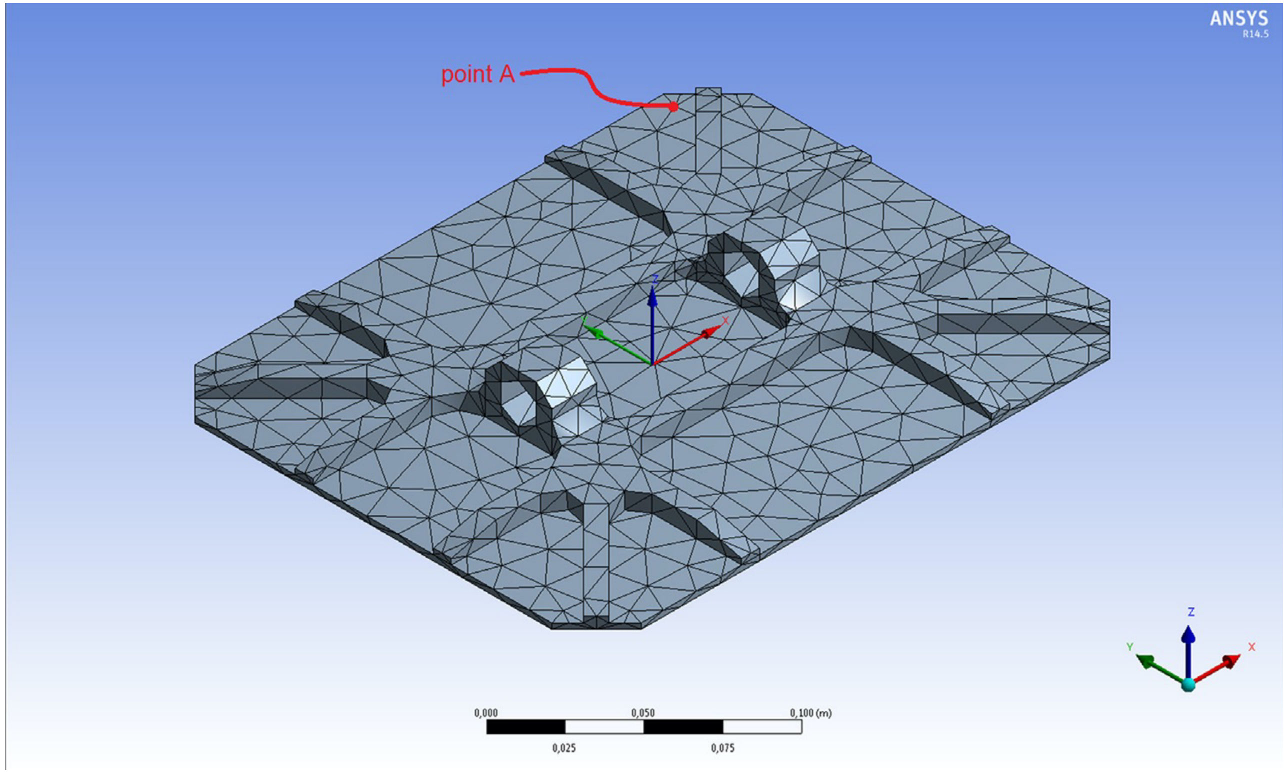
Applications – 3

Mirror plate-shaft assembly



- Mirror plate (original structure) is analyzed
- FRFs of the assembly are calculated by NL structural coupling
- FRFs for different force levels applied at A are compared

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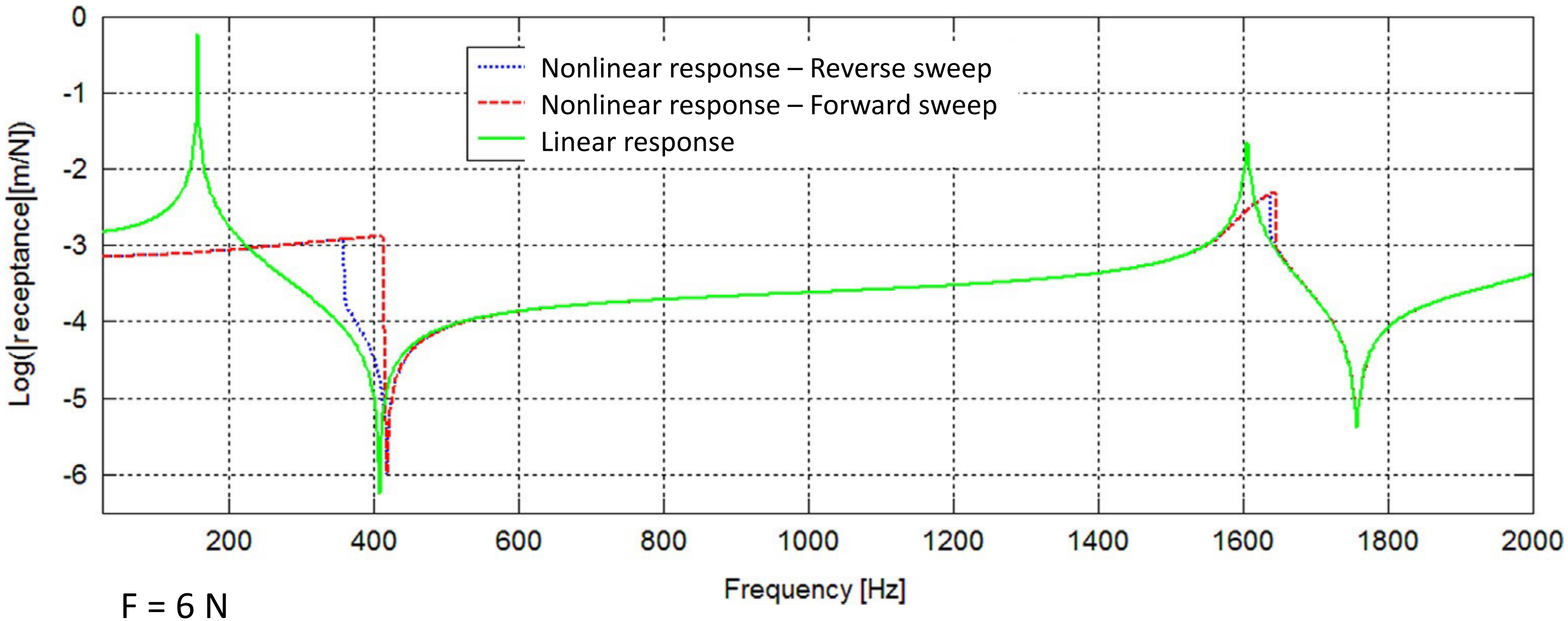
Applications – 3

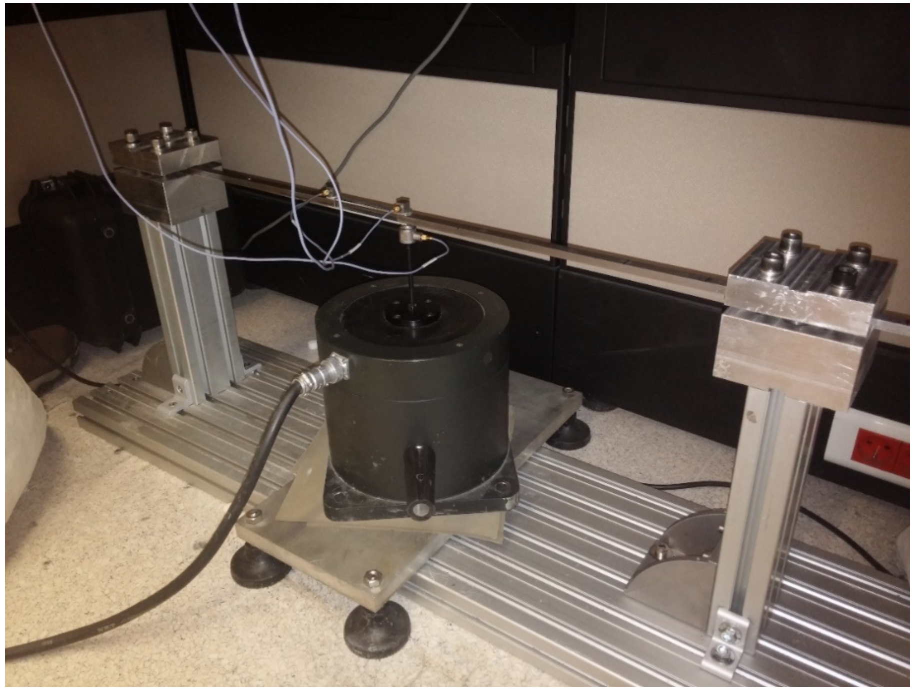
Mirror plate-shaft assembly

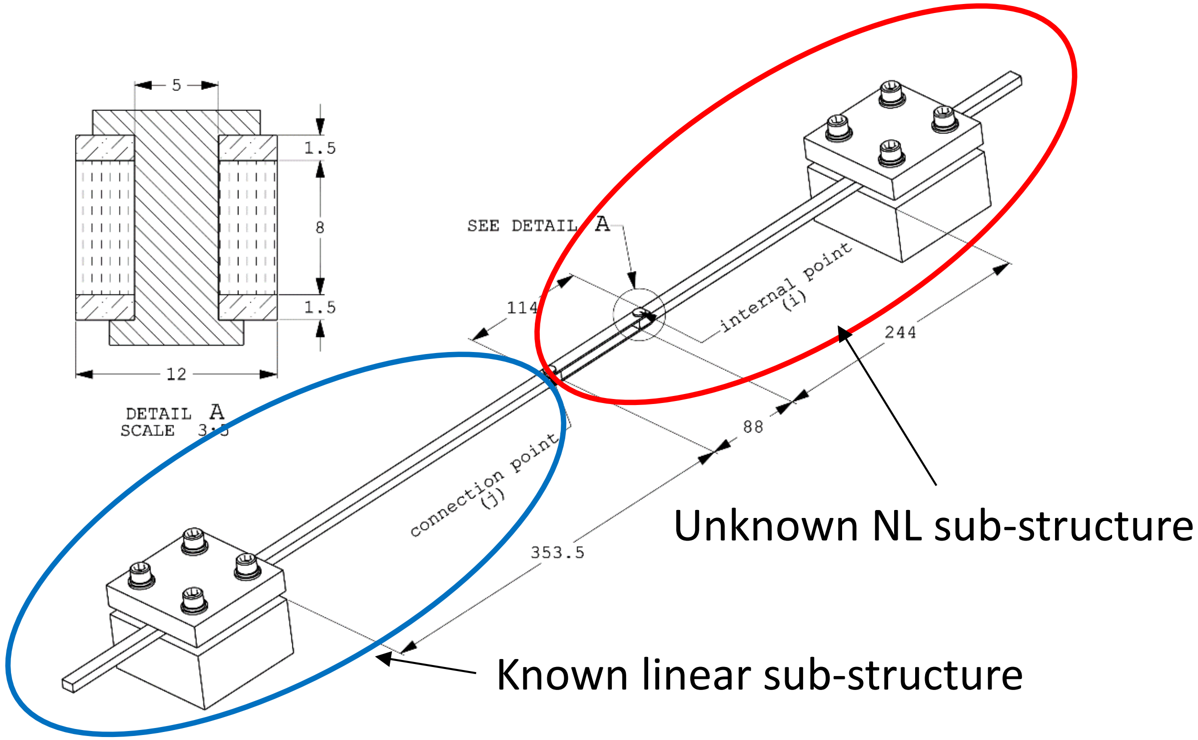


F = 6 N

Applications – 4

Nonlinear Joint Identification by Nonlinear Decoupling*



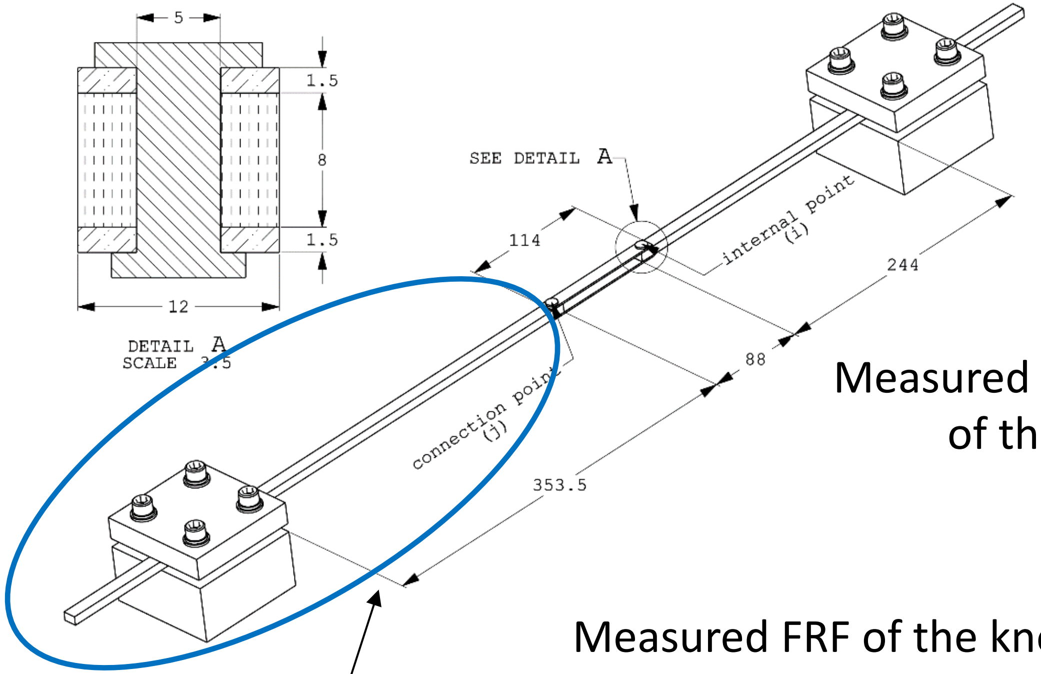


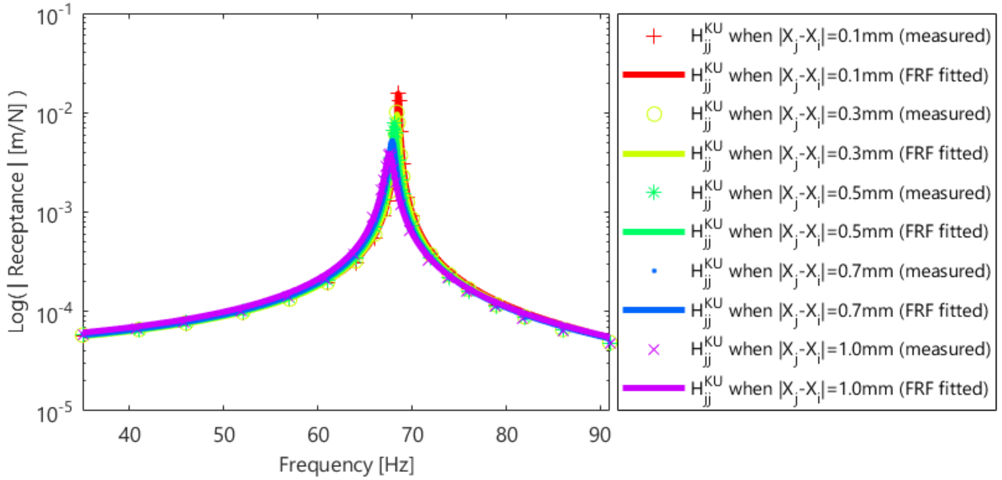
Two cantilever beams connected with a NL element (2 thin beams)

*Kalaycıoğlu, T., Özgüven, H. N., " Experimental Verification of a Recently Developed FRF Decoupling Method for Nonlinear Systems", in: **Dynamics of Coupled Structures**, Vol. 4, Springer, Cham, 2018, pp. 51–63.

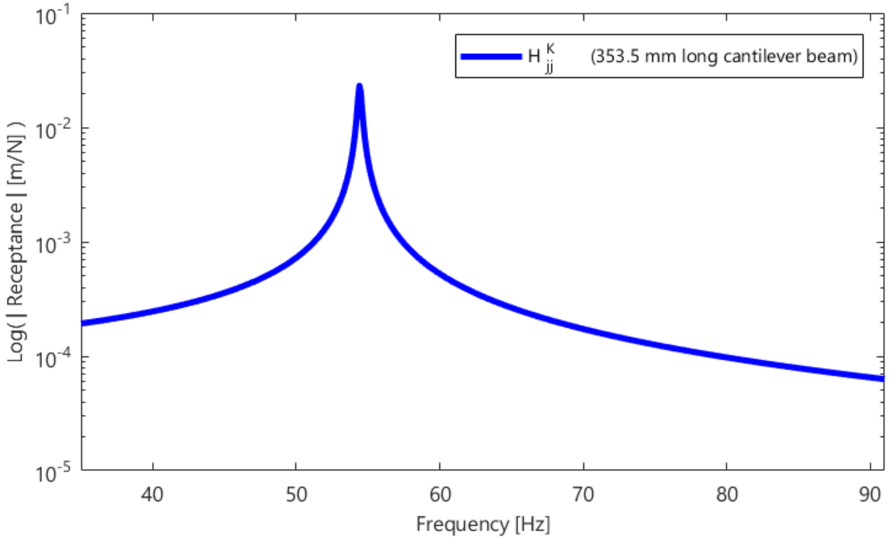
Applications – 4

Nonlinear Joint Identification by Nonlinear Decoupling





Measured quasi-linear FRFs of the assembly

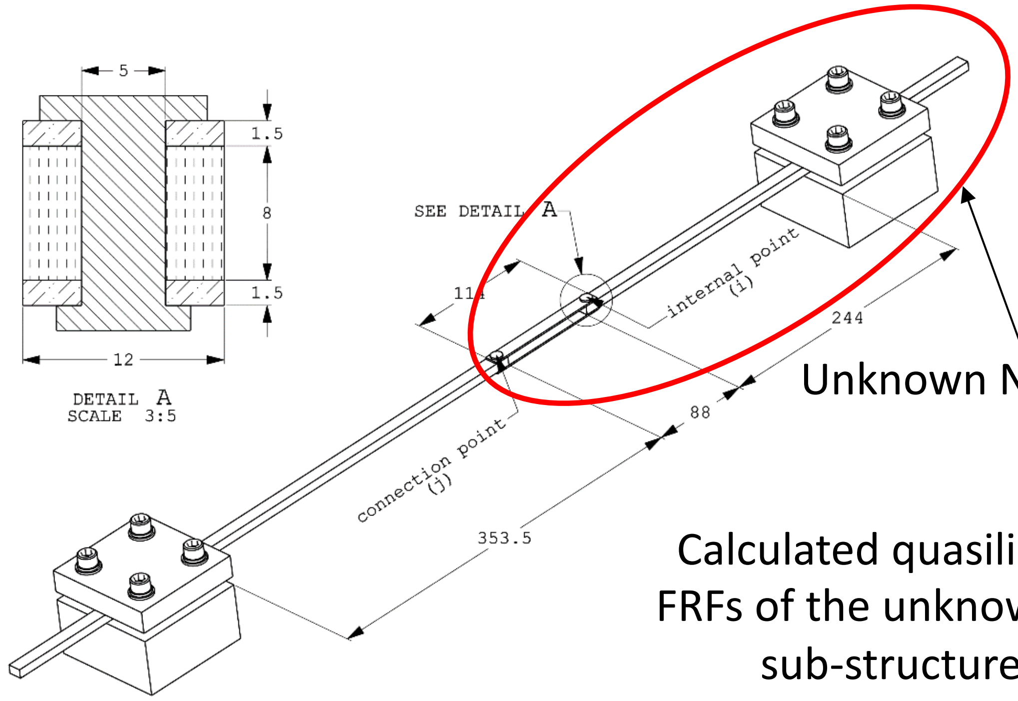


Measured FRF of the known linear structure

Linear sub-structure

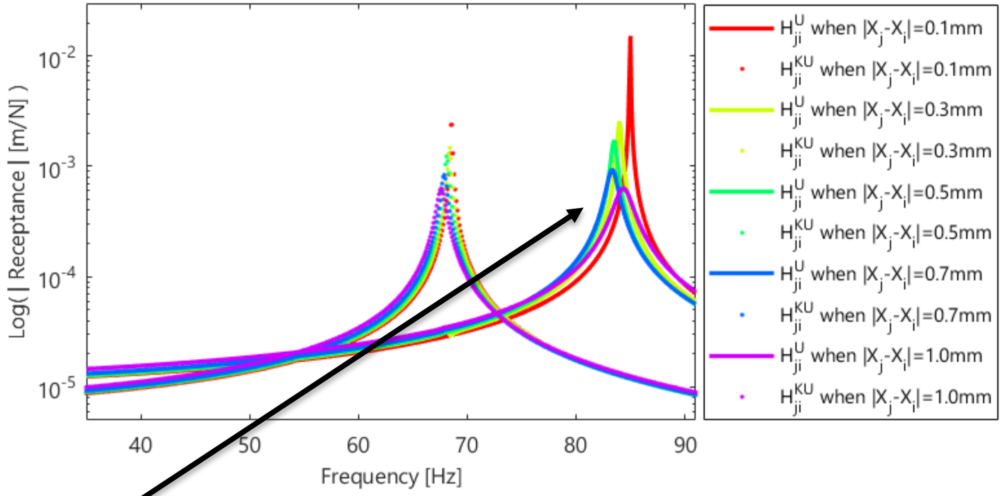
Applications – 4

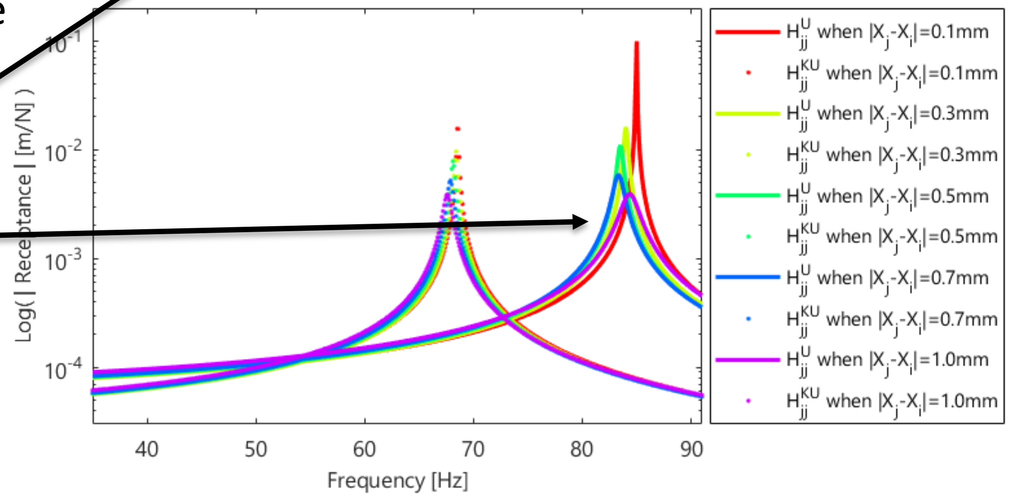
Nonlinear Joint Identification by Nonlinear Decoupling



Unknown NL sub-structure

Calculated quasilinear FRFs of the unknown NL sub-structure





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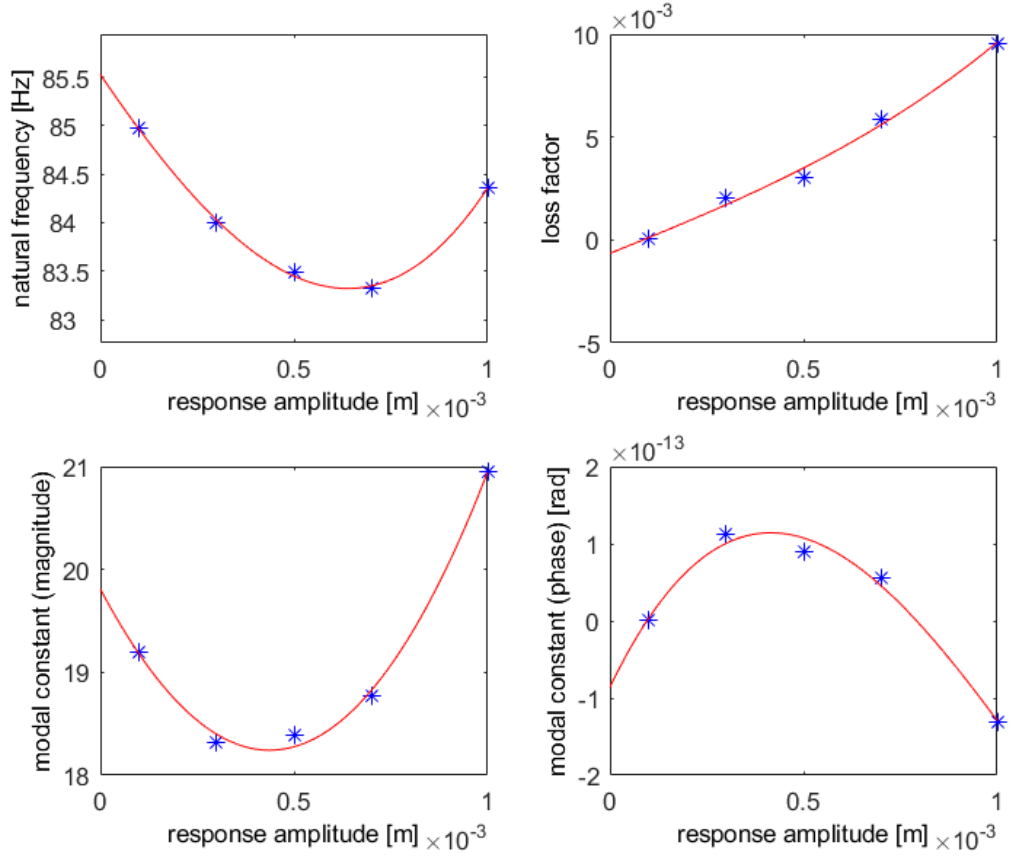
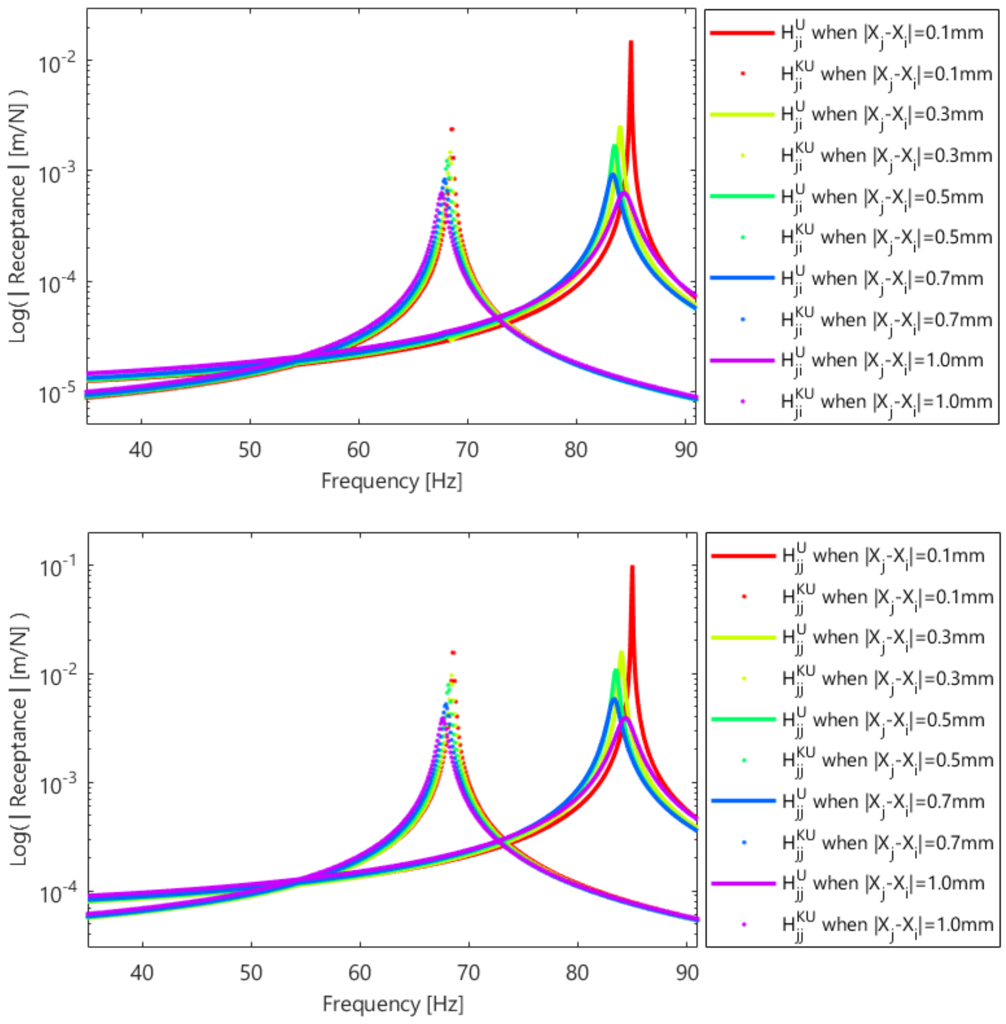
Applications – 4

Nonlinear Joint Identification by Nonlinear Decoupling



Identified modal parameters of unknown NL sub-structure

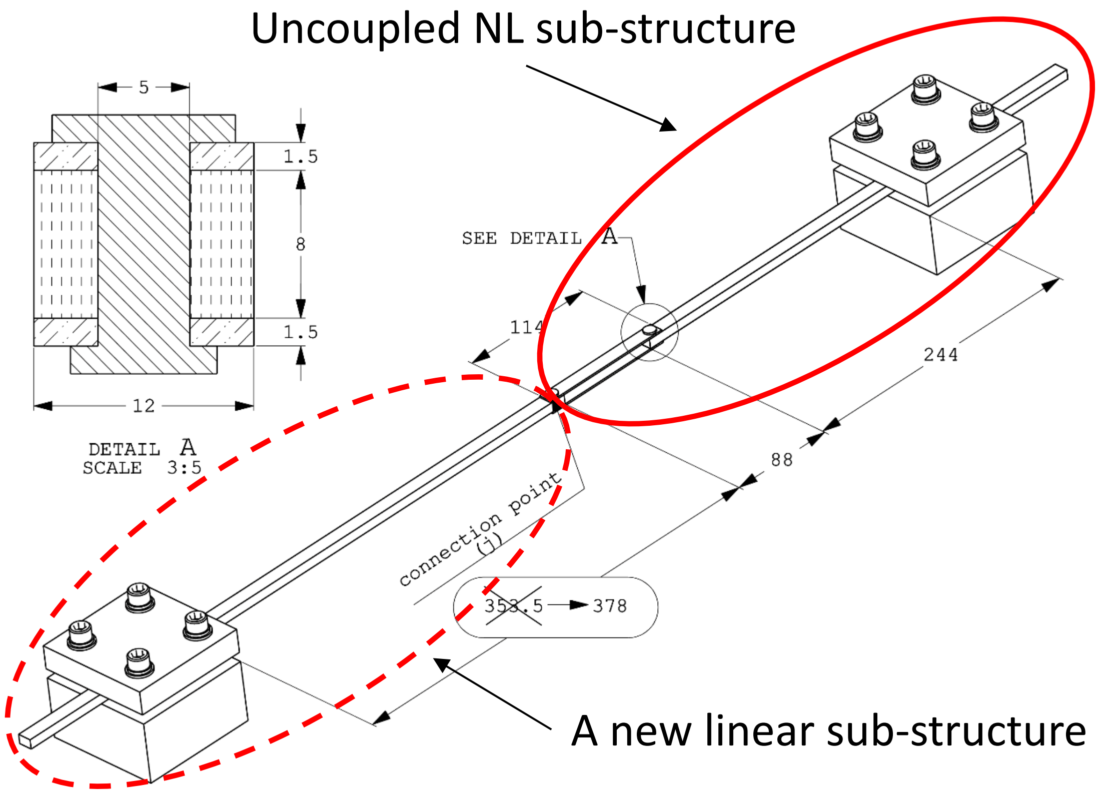
Modal identification

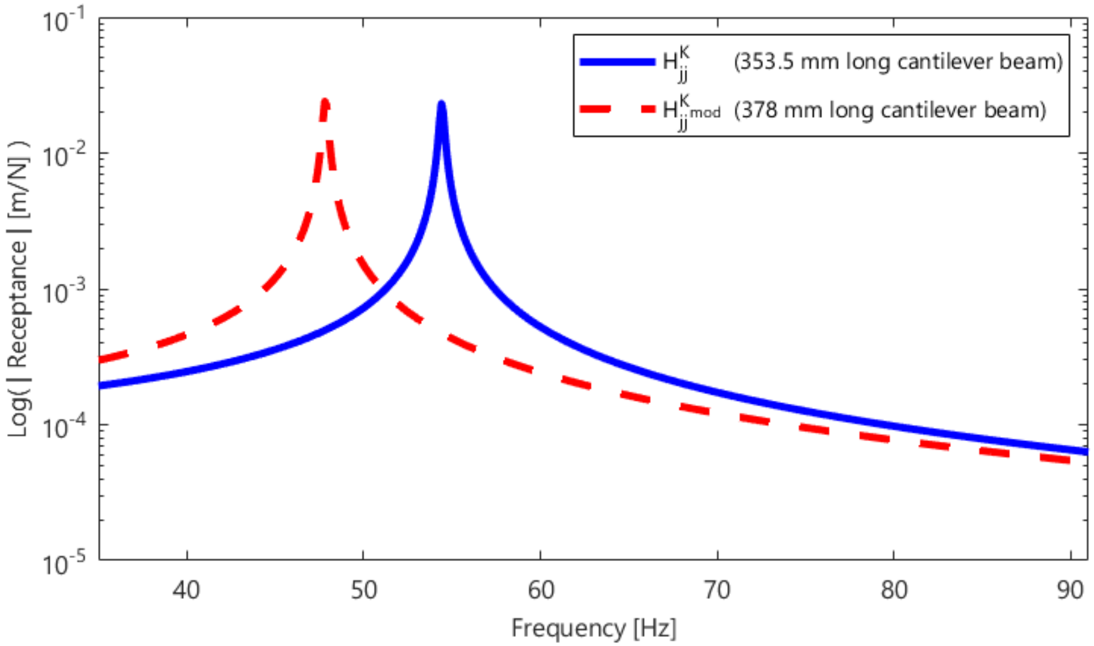


Applications – 5

Verification by Nonlinear Coupling

- Verification is made by coupling the NL sub-structure to a longer beam





Comparison of the FRFs of the original and new linear sub-structures

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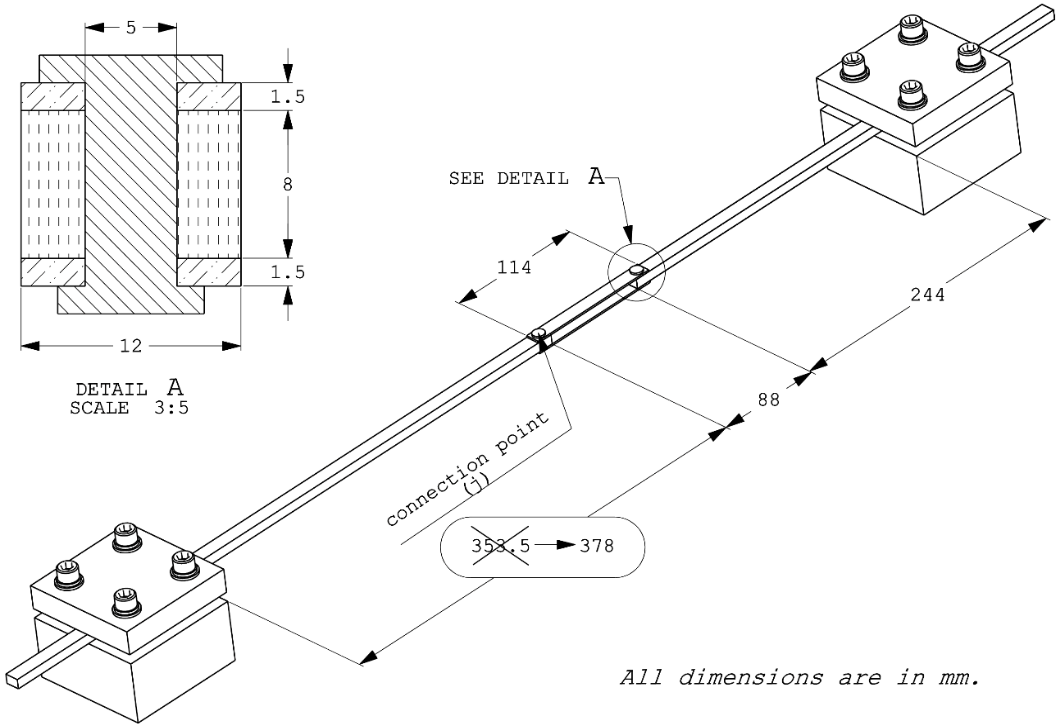
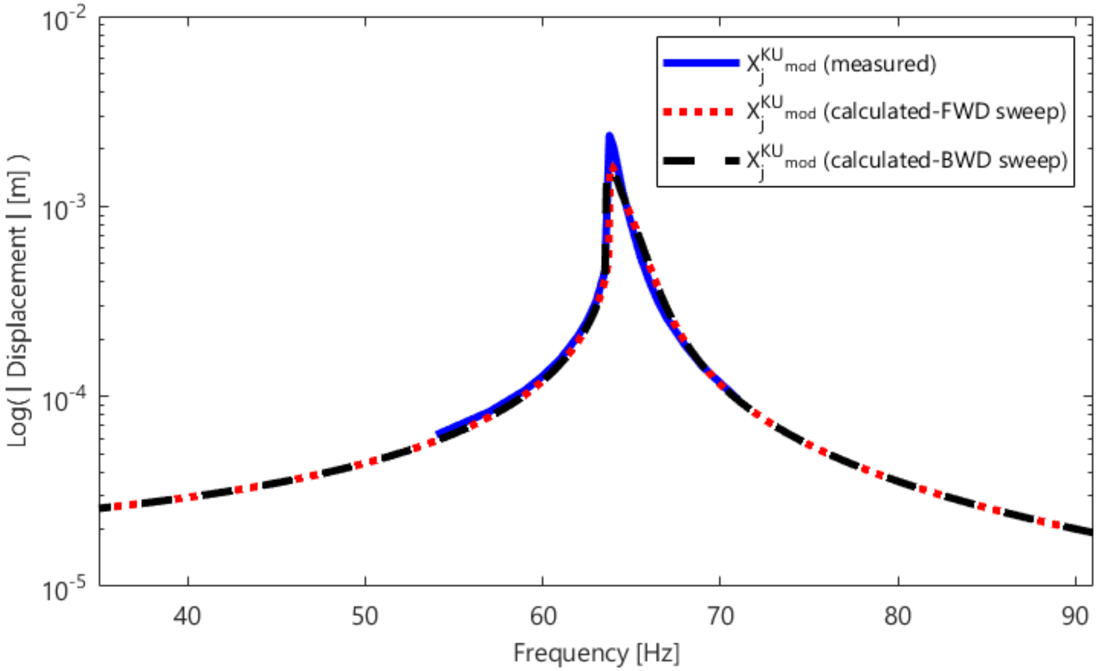
Applications – 5

Verification by Nonlinear Coupling

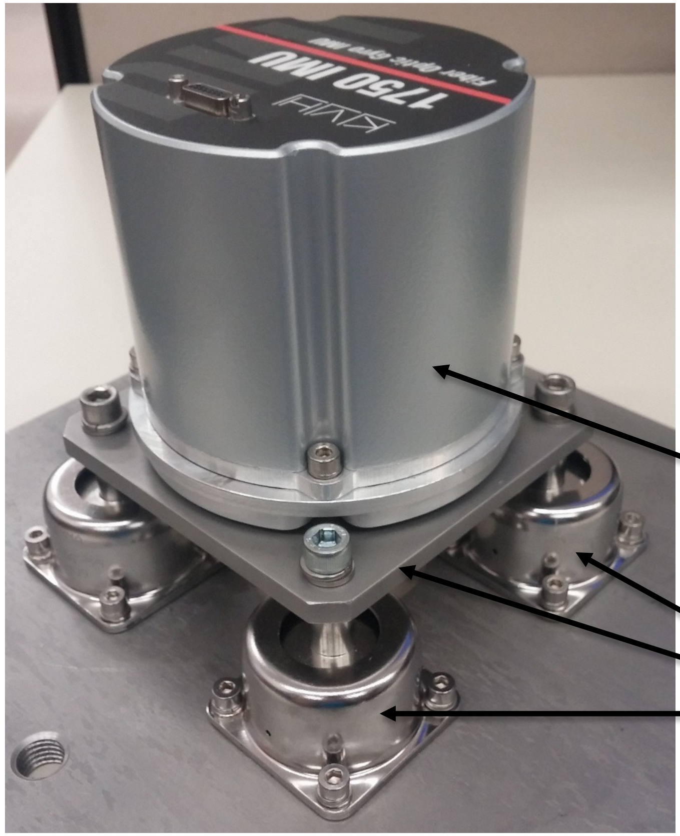




Comparison of the measured and calculated frequency responses of the new NL assembly for a given force

Applications – 6

Nonlinear Identification of Connection Dynamics by Nonlinear Decoupling*



- Inertial Measurement Unit (IMU) and its mechanical interface plate grounded with elastomer isolators
- Used in several aerospace platforms

IMU – Linear known system

Plate and elastomer isolators – NL unknown system

*Kalaycıoğlu, T., "Investigation of Decoupling Techniques for Linear and Nonlinear Systems", **Ph.D. Thesis**, METU, March 2018.

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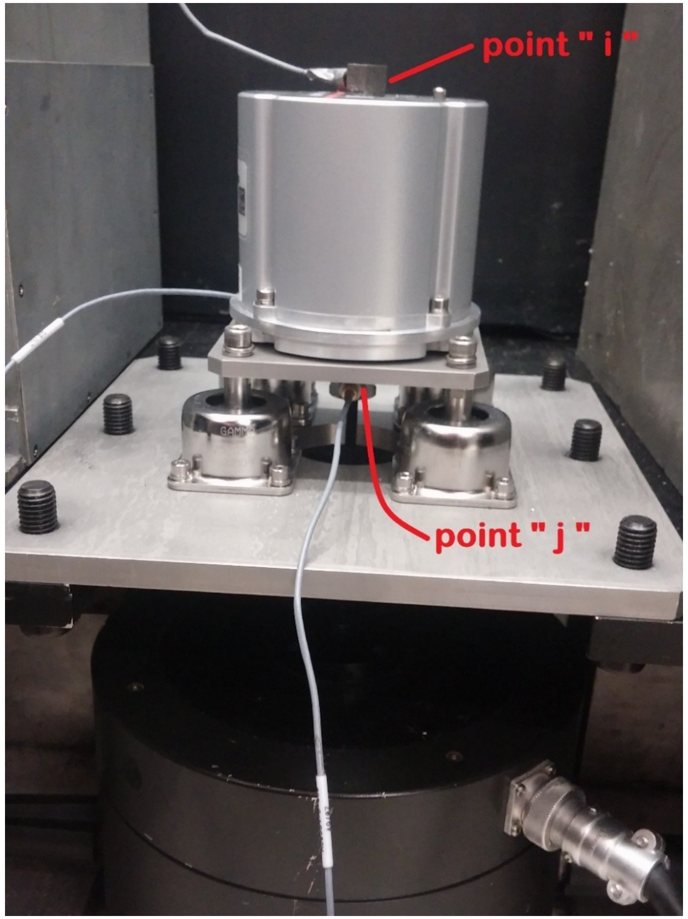
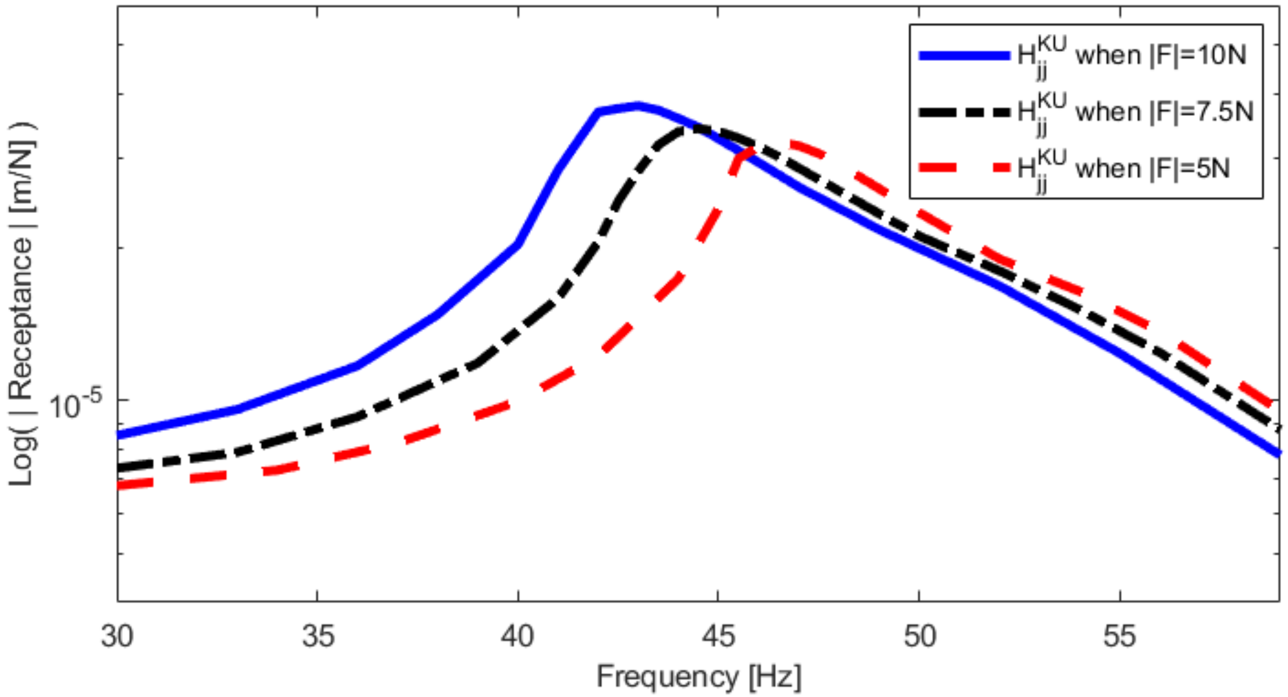
Applications – 6

Nonlinear Identification of Connection Dynamics by Nonlinear Decoupling



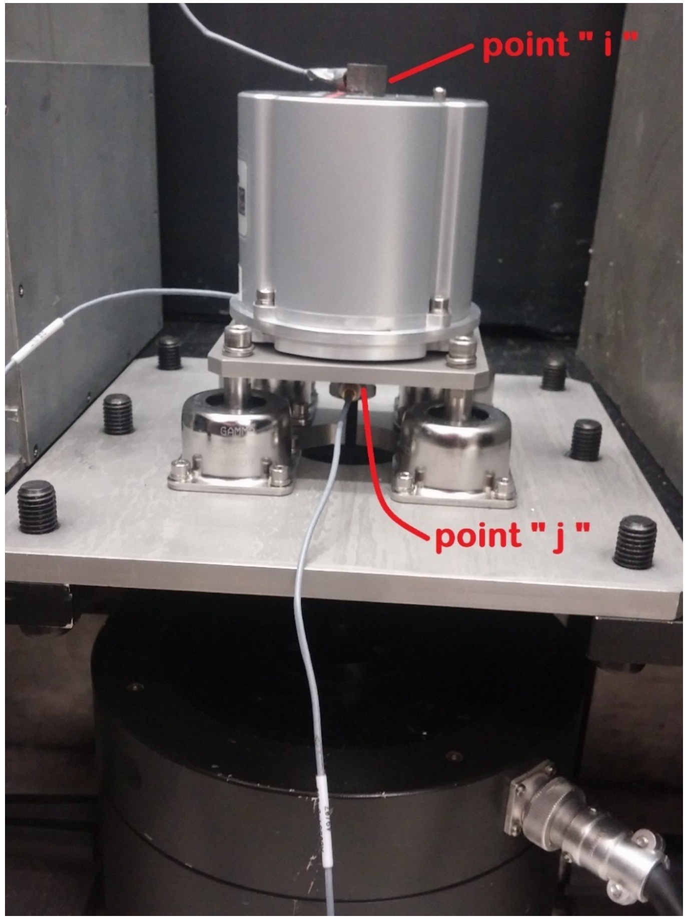


FRFs of coupled system for different forces

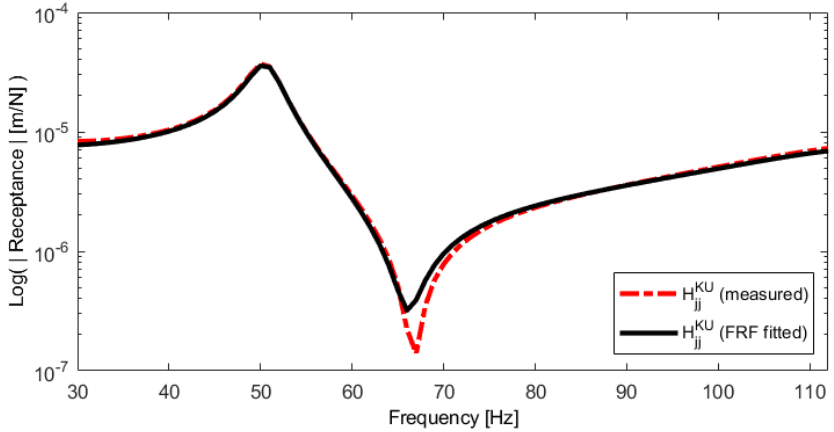
Nonlinearity is clearly observed

Applications – 6

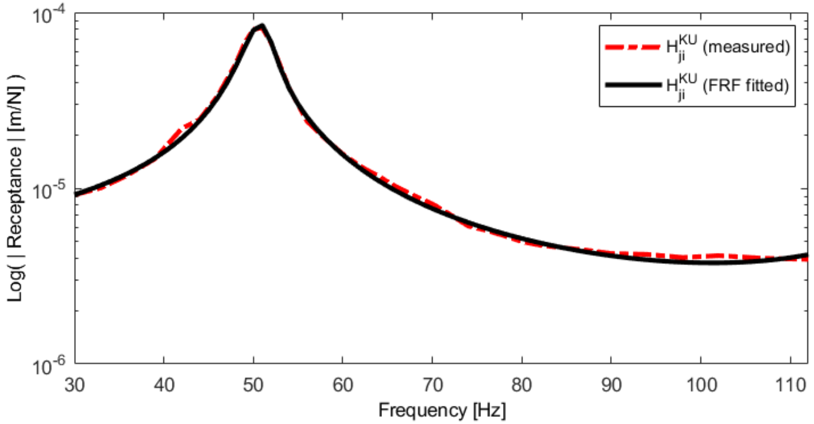
Nonlinear Identification of Connection Dynamics by Nonlinear Decoupling



Y_{jj}^{AB}



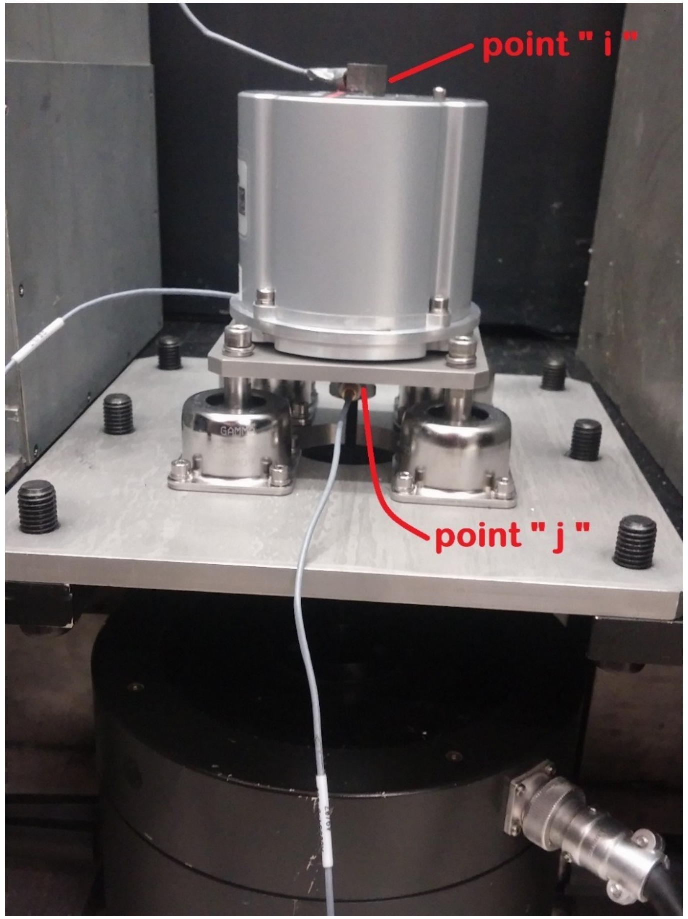
Y_{ji}^{AB}



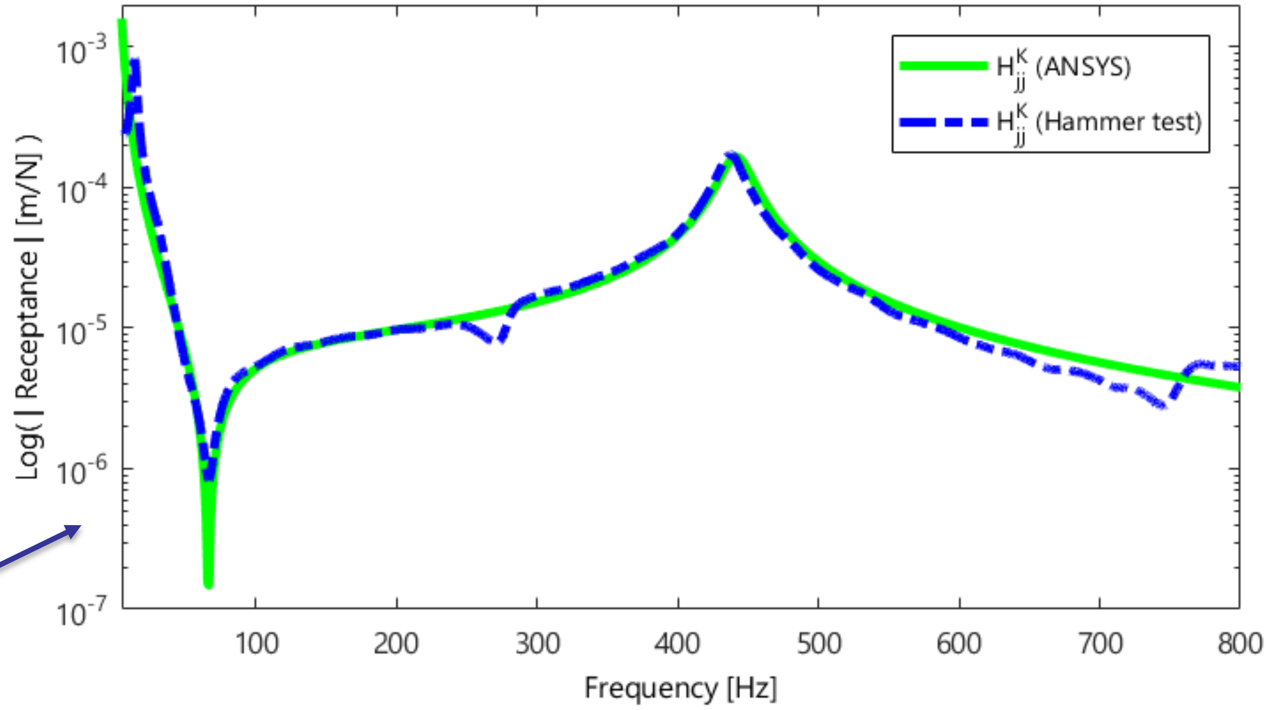
Quasi-linear FRFs of the NL assembly (RCT for U_j =0.05 mm)

Applications – 6

Nonlinear Identification of Connection Dynamics by Nonlinear Decoupling



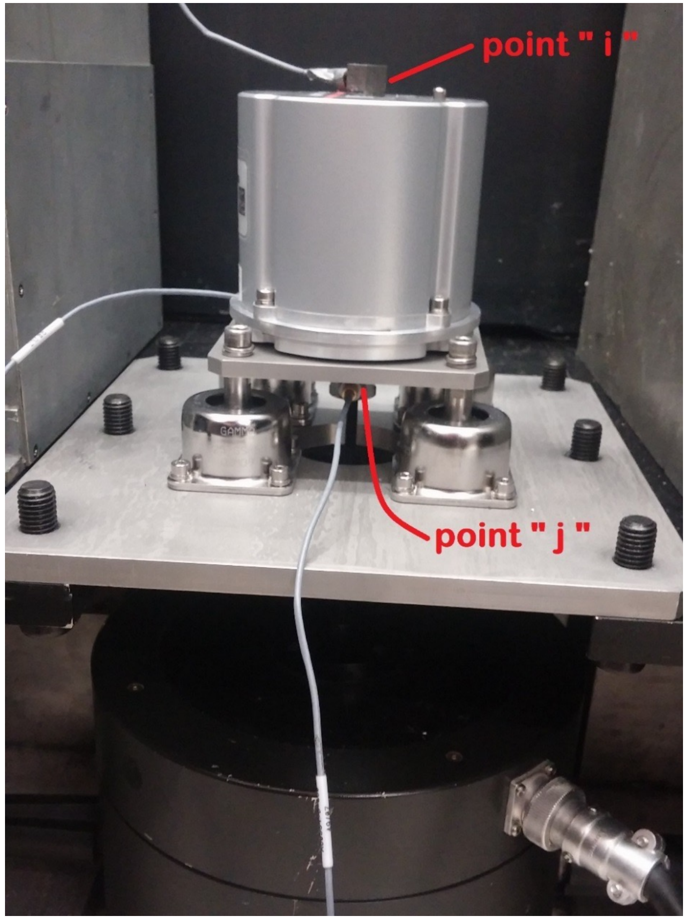
Y_{jj}^{A}



FRF of the known linear sub-system

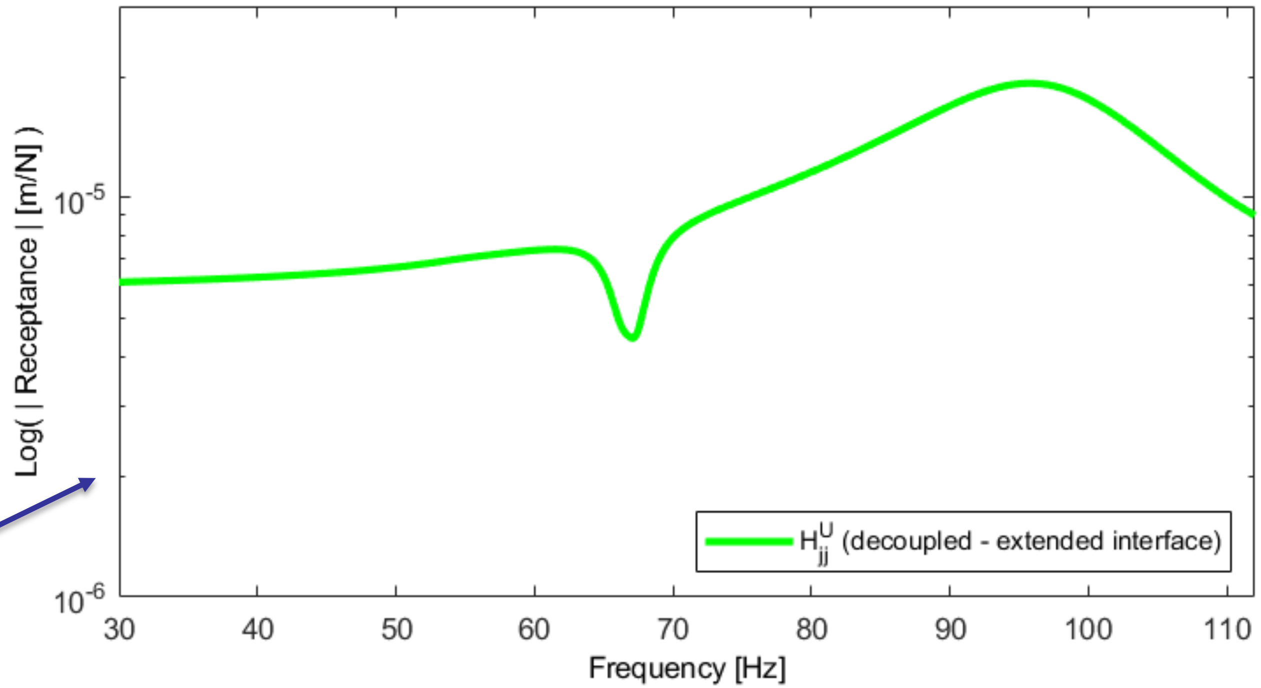
Applications – 6

Nonlinear Identification of Connection Dynamics by Nonlinear Decoupling



- Apply linear decoupling
- Obtain quasilinear FRF of the isolators + plate (for U_j =0.05 mm)

Y_{jj}^{B}



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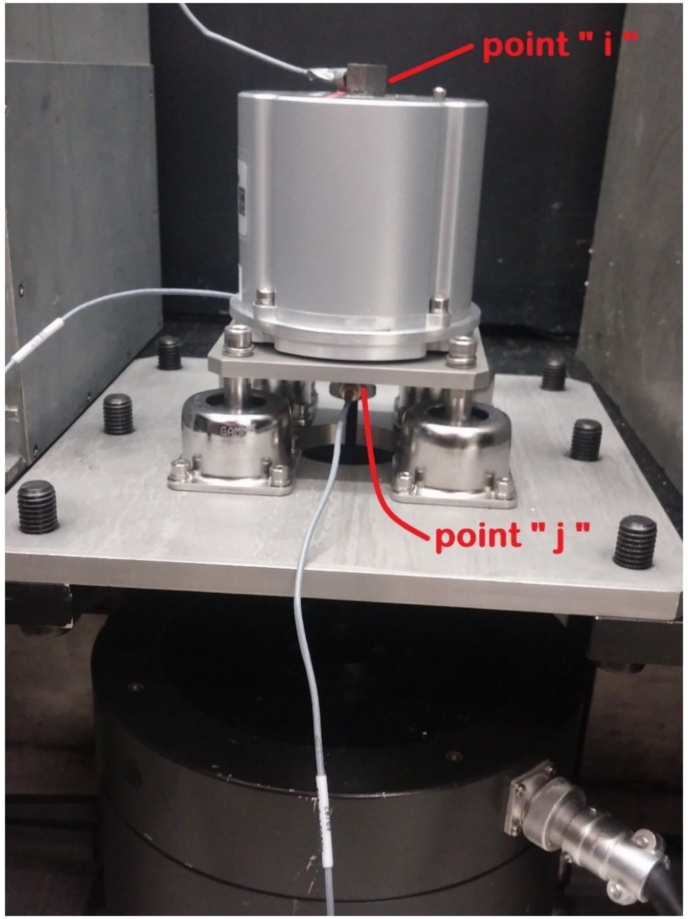
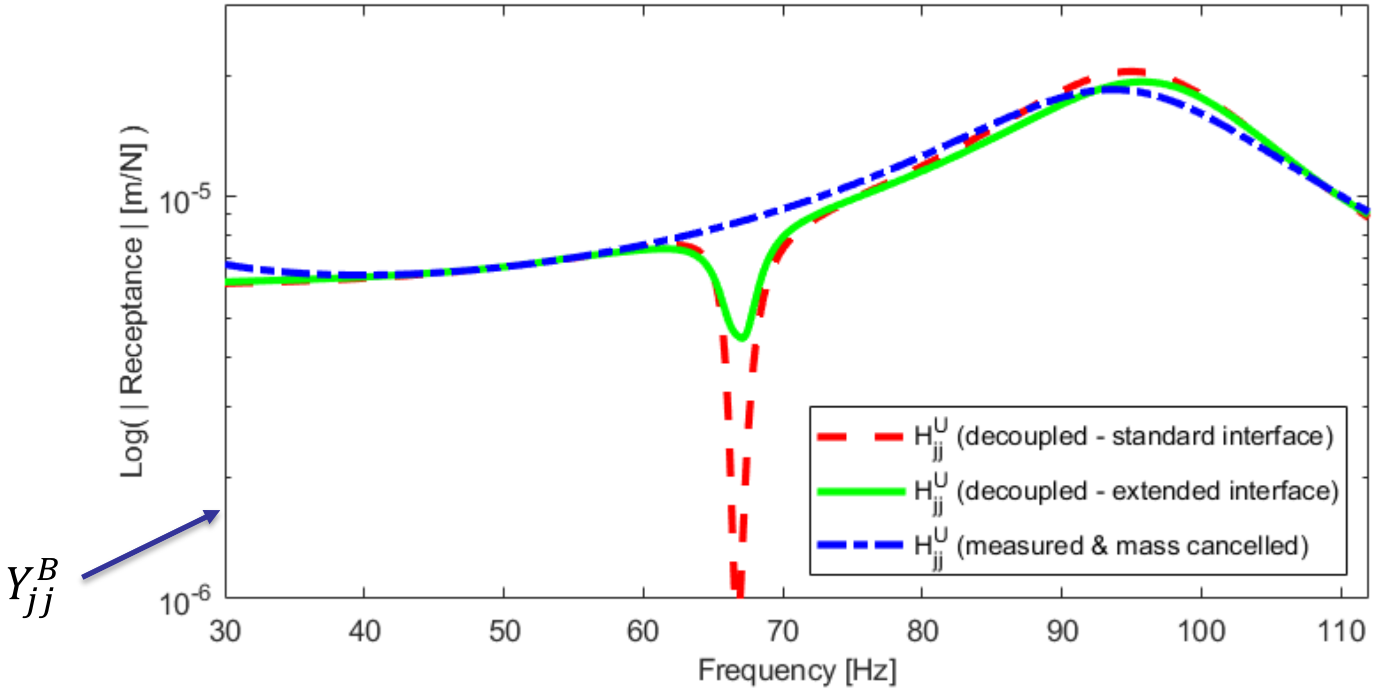
Applications – 6

Nonlinear Identification of Connection Dynamics by Nonlinear Decoupling

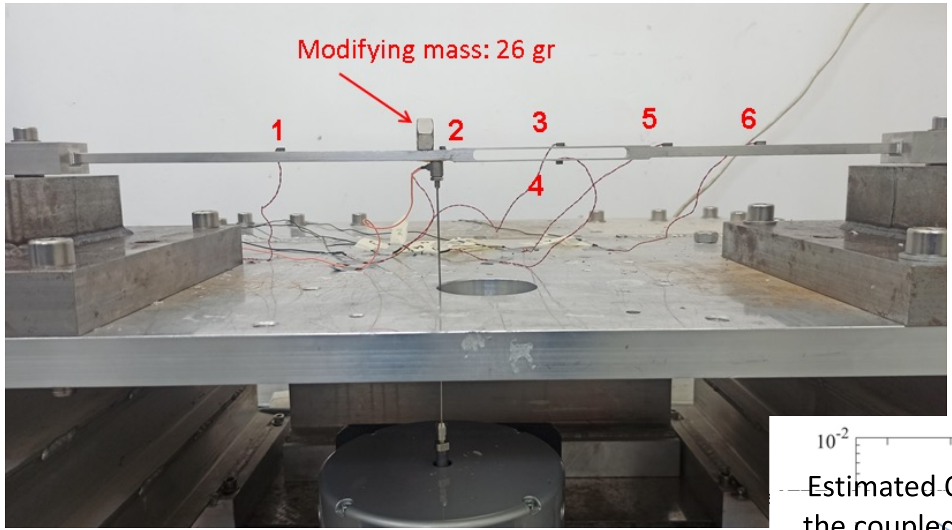


Comparison of calculated and measured quasi-linear FRFs of the NL sub-system

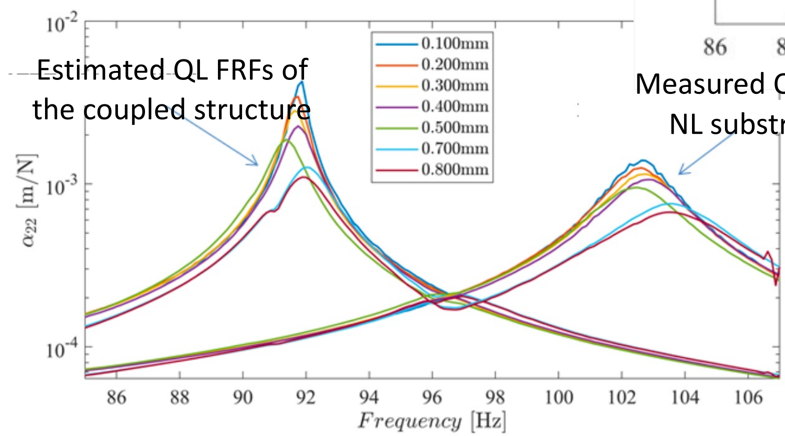


Applications – 7

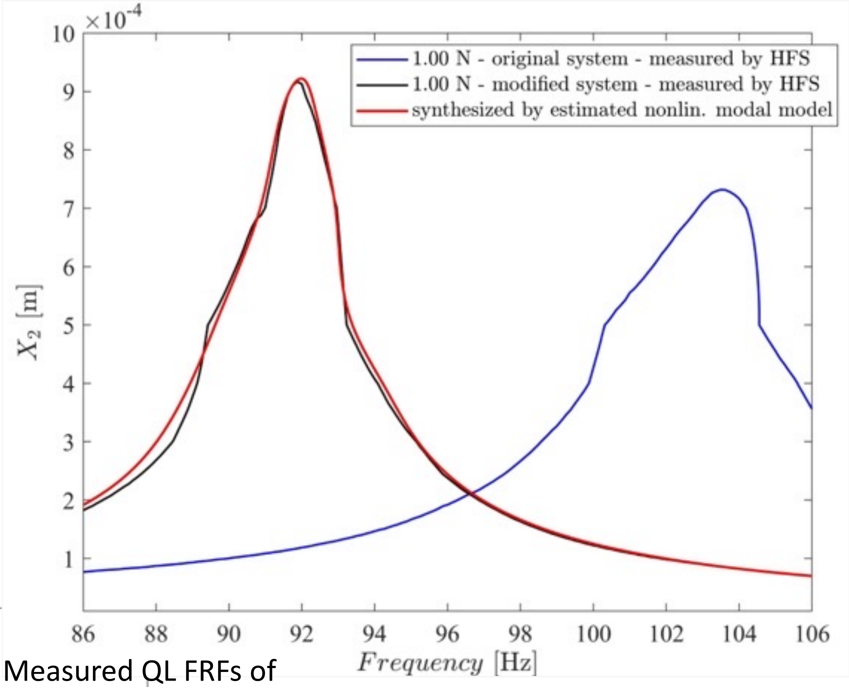
Nonlinear Coupling*



Estimated QL FRFs of the coupled structure



Measured QL FRFs of NL substructure



*Ekinci, C., et. al, "Extension and Experimental Verification of an Efficient Re-Analysis Method for Modified Non-linear Structures", **IMAC 42**, 2024.

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Final Remarks

- Main assumption: Harmonic excitation yields harmonic response in the frequency range of interest.
- Good news: This assumption is valid in a wide range of industrial applications.
- Quasi-linearization: Nonlinear structures depict some features of linear structures under certain conditions.
- Quasi-linearization in analytical computations: By using Nonlinearity Matrix calculated using Describing Functions.
- Quasi-linearization in experimental analysis: By using RCT (response-controlled stepped-sine testing).
- Quasi-linearization makes it possible to extend the structural coupling/decoupling methods for L systems to NL systems
- Nonlinear sub-structuring methods presented are frequency-based, and use quasi-linear behavior of NL systems.
- Limitations: There are limitations on the application of some of the methods.
- Applications: All methods are successfully applied on real systems.

  Some are applied even on complex engineering systems.